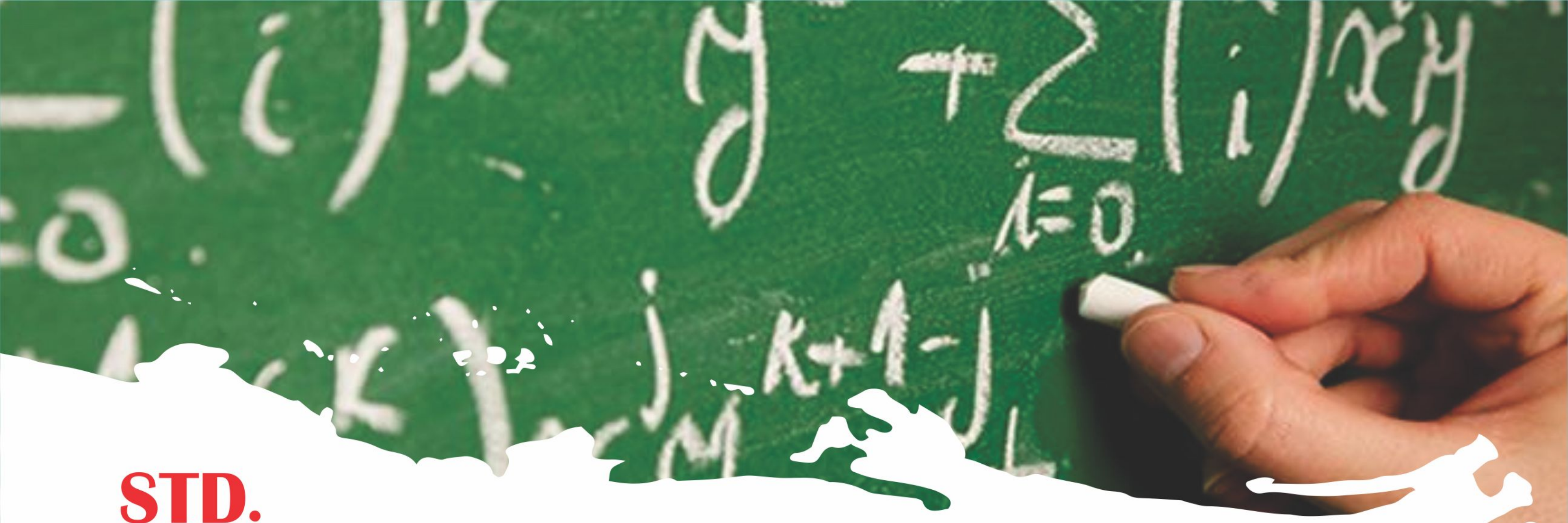

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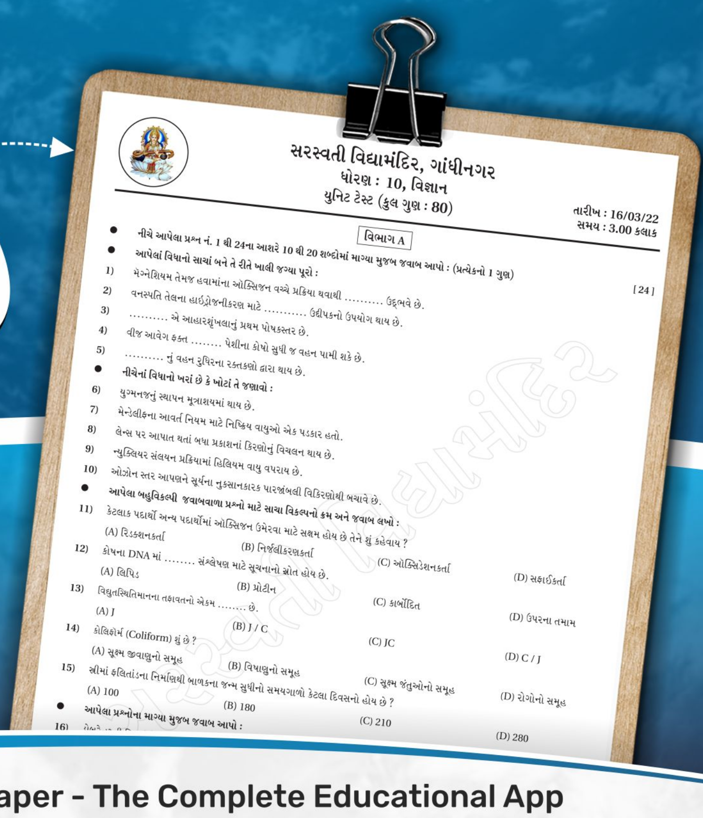

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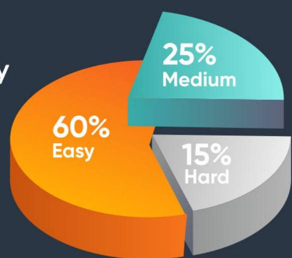

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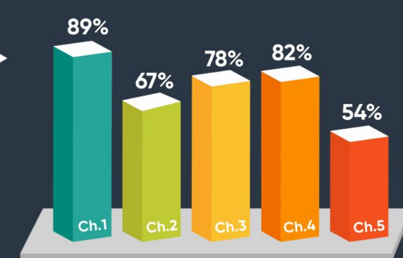

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2 Polynomials

Kumar

2.1 - Things To Remember S1

❖ **Algebraic Polynomial :**

➤ In algebraic polynomial, a variable and constant connect with addition subtraction, multiplication or division.

Ex. $2x + 3$, $5 - 7x$, $\frac{2x}{3}$ etc.

❖ **One Variable Polynomial :**

1) **Variable :** A symbol which takes different values is called variable. It is denoted by x, y, z ...

2) **Constant :** A symbol which does not take different values is called constant. It is denoted by $a, b, c, 2, 3, -5, 7, \pi$ etc.

3) **Algebraic Expressions :**

➤ Algebraic expression is of the form of multiplication of variable and constant.

Algebraic Expressions = Constant × Variable

$= 5 \times x$

$= 5x$

➤ In algebraic polynomial there is whole number as the exponents of the variable.

Ex. In $5x^2$, The exponent of variable x is 2, the whole number.

➤ The exponent of variable in algebraic expression is not negative integer.

Ex. $x + \frac{1}{x} = x + x^{-1}$ Here the exponent of x is -1. So this algebraic expression is not a polynomial.

➤ The exponent of variable in algebraic expression is not fraction.

Ex. $\sqrt{x} + 3 = x^{\frac{1}{2}} + 3$ Here the exponent of x is $\frac{1}{2}$, so this algebraic expression is not a polynomial.

➤ If the algebraic expression is a polynomial then the exponent of variable is whole number. It is never negative integer or fraction.

4) **Polynomials in one variable :** A polynomial with one variable is called one variable polynomial.

Ex. $32x^2 - 5x$ is a polynomial with x variable

$3y^2 + 5y$ is a polynomial with y variable.

$t^2 + 4$ is a polynomial with t variable.

5) **Terms of Polynomial :** Terms of the polynomial is a variable with some constant and with the exponent of variable the whole number.

Ex.

➥ In the polynomial $x^2 + 2x$, x^2 and $2x$ are terms.

➥ In the polynomial $3y^2 + 5y + 7$, $3y^2$, $5y$ and 7 are terms.

➥ In the polynomial $-x^3 + 4x^2 + 7x - 2$, there are four terms $-x^3$, $4x^2$, $7x$ and -2.

➤ In the variable in polynomial is x, we may denote the polynomial by $p(x)$ or $q(x)$ or $r(x)$ etc.

Ex. $p(x) = 2x^2 + 5x - 3$

$q(x) = x^3 - 1$

$r(y) = y^3 + y - 1$

6) **Constant polynomial :** 2 is a constant polynomial 2, –5, 7 are constant polynomial. The degree of a non zero constant polynomial is zero.

7) **Zero polynomial :** constant polynomial 0 (zero) is called zero polynomial. The degree of zero polynomial is not defined.

8) **Type of polynomials according to their number of terms :**

➤ **Monomials :**

A polynomial having only one term is called monomials. Ex. $p(x) = 5x$

➤ **Binomials :**

A polynomial having only two term is called binomial. Ex. $p(x) = 5x + 3$

➤ **Trinomials :**

A polynomial having only three term is called trinomials.

Ex. $p(x) = x^2 + x + \pi$

$q(y) = y^4 + y - 5$

9) **Degree of the polynomial :**

The highest power of the variable in a polynomial is called the degree of the polynomial.

Ex. $x^5 - 2x^3 + x$ The highest power of the variable x is 5. So the degree of the polynomial is 5.

2, 2 is written as $2x^0$. The highest power of the variable x is zero.

$2 - y^2 - y^3 + 2y^8$. The highest power of the variable y is 8. So the degree of the polynomial is 8.

10) **Type of polynomial on the basis of degree :**

Linear polynomial : A polynomial of degree one is called a linear polynomial.

$p(x) = ax + b$, Where $a \neq 0$, a, $b \in$ R.

$p(y) = ay + b$, Where $a \neq 0$, a, $b \in$ R.

$ay + b$ is a linear polynomial with y variable example, $p(x) = 3x - 6$, $p(x) = 7x$

Quadratic polynomial : A polynomial of degree two is called a quadratic polynomial.

$p(x) = ax^2 + bx + c$, where $a \neq 0$, a, b, $c \in$ R

$p(x) = x^2 + 5x + 10$, $p(x) = 3x^2 - 11$,

$p(x) = \frac{3x}{7} - x^2$

Cubic polynomial :

A polynomial of degree three is called a cubic polynomial.

$p(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$, a, b, c, $d \in$ R.

$p(x) = x^3 - 4x^2 + 7$

$p(x) = x^3 + x^2 + x + 1$

11) **Number of terms on the basic of power of variable in the polynomial :**

(1) A quadratic polynomial in one variable will have at most 3 terms.

(2) A cubic polynomial in one variable will have at most 4 terms.

(3) A linear polynomial in one variable will have at most 2 terms.

12) **Polynomial with more than one variables :** A polynomial in more than one variable is called a polynomial with more than one variable example.

$x^2 + y^2 + xyz$ where x, y, z are variables.

It is a polynomial with three variables

$p^2 + q^{10} + r$ where p, q and r are variable.

It is a polynomial with three variables

13) **Standard (General) form of polynomial :** A polynomial whose terms are ordered from biggest exponent to lowest exponent is called the standard form of polynomial. Example,

$p(x) = 12x + 5x^3 + 8 - 6x^2$

Standard form of $p(x)$ is

$p(x) = 5x^3 - 6x^2 + 12x + 8$

➥ **First term :** when the polynomial is written in the standard form the term with the biggest exponent is called the leading term or first term in $p(x) = 5x^3 - 6x^2 + 12x + 8$, $5x^3$ is the first term.

➥ **Coefficient of first term :** When the polynomial is written in the standard form, the constant of the first term is called coefficient of first term in

$p(x) = 5x^3 + 6x^2 + 12x + 8$.

5 is the co-efficient of first term.

Remember :

$p(x) = \frac{x}{2} - 3x^2 + \frac{5}{2}x^3 - 5x^4$

Term	Coefficient	exponent
$\frac{x}{2}$	$\frac{1}{2}$	1
$-3x^2$	-3	2
$\frac{5}{2}x^3$	$\frac{5}{2}$	3
$-5x^4$	-5	4

Exercise - 2.1 S2

1) Which of the following expressions are polynomials in one variable and which are not ? State reasons for your answer.

(i) $4x^2 - 3x + 7$ #

➤ $4x^2 - 3x + 7x^0$

The exponents of variable x are whole number.

∴ $4x^2 - 3x + 7$ is a polynomial with one variable.

(ii) $y^2 + \sqrt{2}$

➤ $y^2 + \sqrt{2}y^0$:

Here, the exponents of variable y are whole number.

∴ $y^2 + \sqrt{2}y^0$ is a polynomial with one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

➤ $3t^{\frac{1}{2}} + \sqrt{2}t$

Here the exponent of $t^{\frac{1}{2}}$ is $\frac{1}{2}$.

It is not a whole number.

∴ $3\sqrt{t} + t\sqrt{2}$ is not a polynomial.

(iv) $y + \frac{2}{y}$

➤ $y + 2y^{-1}$

Here the exponent of y^{-1} is –1.

It is not a whole number.

∴ $y + \frac{2}{y}$ is not a polynomial.

(v) $x^{10} + y^3 + t^{50}$

➤ Here, the exponents of x, y and z are whole numbers.

∴ $x^{10} + y^3 + t^{50}$ is a polynomial with three variables x, y and t.

2) Write the coefficients of x^2 in each of the following :

(i) $2 + x^2 + x$

➤ The coefficient of x^2 is 1.

(ii) $2 - x^2 + x^3$

➤ The coefficient of x^2 is –1.

(iii) $\frac{\pi}{2}x^2 + x$

➤ The coefficient of x^2 is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x - 1$

➤ $\sqrt{2}x - 1$

$\sqrt{2}x - 1 + 0 \cdot x^2$

The coefficient of x^2 is 0.

UID : P1-C2-S2-Q1.1

3) Give one example each of a binomial of degree 35 and of a monomial of degree 100.

➤ A binomial of degree $3x^{35} - 14$

➤ A monomial of degree $\sqrt{3}y^{100}$

Note : You can write some more polynomials with different Coefficients.

4) Write the degree of each of the following polynomials :

(i) $5x^3 + 4x^2 + 7x$

➤ The highest power of the variable x is 3. So the degree of the polynomial is 3.

(ii) $4 - y^2$

➤ The highest power of the variable y is 2 so the degree of the polynomial is 2.

(iii) $5t - \sqrt{7}$

➤ The highest power of the variable t is 1. So the degree of the polynomial is 1.

(iv) 3

➤ $3 = 3x^0$

The highest power of the variable x is 0. So the degree of the polynomial is 0.

5) Classify the following as linear, quadratic and cubic polynomials :

(i) $x^2 + x$

➤ The degree of $x^2 + x$ is 2. So it is a quadratic polynomial.

(ii) $x - x^3$

➤ The degree of $x - x^3$ is 3.

So it is a cubic polynomial.

(iii) $y + y^2 + 4$

➤ The degree of $y + y^2 + 4$ is 2. So it is a quadratic polynomial.

(iv) $1 + x$

➤ The degree of $1 + x$ is 1. So it is a linear polynomial.

(v) $3t$

➤ The degree of $3t$ is 1. So it is a linear polynomial.

(vi) r^2

➤ The degree of r^2 is 2.

So it is a quadratic polynomial.

(vii) $7x^3$

➤ The degree of $7x^3$ is 3.

So it is a cubic polynomial.

Practice Work S12

1.1) Which of the following expressions are polynomials in one variable and which are not ? State reasons for your answer.

(i) $3x^2 + 4x + 5$ # (iv) $z + \frac{3}{z}$

(ii) $y^2 + \sqrt{7}$ (v) $x^{25} + y^{20} + z^8$

(iii) $5\sqrt{p} + p\sqrt{5}$

1.2) Write the coefficients of x^2 in each of the following.

(i) $5 - x^2 + x$ (iii) $\sqrt{3}x - 1$

(ii) $8 + x^2 + x$ (iv) $\frac{\pi}{2}x^2 + x$

1.3) Give one example each binomial degree 25 a monomial degree 110

1.4) Write the degree of each the polynomials.

(i) $6x^3 - 4x^2 + 2$ (iii) $18t - \sqrt{5}$

(ii) $8 - y^2$ (iv) 125 [**Hints :** $125x^0$]

1.5) Classify the following as linear quadratic and cubic polynomials.

(i) $x^2 + 1$ (iv) $5 + x$ (vii) r^2

(ii) $2x - x^3$ (v) $5x$

(iii) $y + y^2 + 8$ (vi) r^3

Answers

1.1) **(i)** One variable polynomial **(ii)** One variable polynomial **(iii)** Not a polynomial as the exponent of p is $\frac{1}{2}$ **(iv)** Not a polynomial as the exponent of z is – 1 **(v)** Polynomial with three variables.

1.2) **(i)** (–1) **(ii)** 1 **(iii)** 0 **(iv)** $\frac{\pi}{2}$

1.3) $8x^{25} - 3$, $\sqrt{7}y^{110}$ [**Note :** You can write some more polynomials with different coefficients]

1.4) **(i)** 3 **(ii)** 2 **(iii)** 1 **(iv)** 0

1.5) **(i)** Linear **(ii)** cubic **(iii)** quadratic **(iv)** Linear **(v)** Linear **(vi)** cubic **(vii)** quadratic

2.2 - Things To Remember S1

❖ **Zeroes of a polynomial :**

1) **Zeroes of a polynomial :** A real number a is a zero of a polynomial $p(x)$ if $p(a) = 0$

In this case, a is also called a root of the equation $p(x) = 0$.

Ex. $p(x) = x - 3 = 0$

$\therefore x = 3$

3 is the roots of the equation $p(x) = 0$
3 is a zero of the polynomial $p(x)$.

➤ 3 is a zero of $p(x)$ so $p(3) = 0$.

➤ To find zero of $p(x) = x - 3$, take $x - 3 = 0$ $x = 3$.

➤ $p(x) = 0$ is an equation and 3 is the root of the equation $p(x) = 0$

➤ A non zero constant polynomial has no zero.

➤ Every real number is a zero of the zero polynomial.

2) **Some important result for zeroes of a polynomial :**

➤ 0 may be the zero of a polynomial but the zeroes of a polynomial need not zeroes be 0.

➤ Every linear polynomial in one variable has a unique zero.

Ex. zero of $x + 3$ is – 3

zero of $2x + 7$ is $\frac{-7}{2}$

zero of $x - 3$ is 3

➤ A polynomial except linear has one or more than one zeroes.

UID : P1-C2-S12-Q1.1.1♦

Exercise - 2.2

S3

1) Find the value of the polynomial $5x - 4x^2 + 3$ at :

(i) $x = 0$ #

➤ $p(x) = 5x - 4x^2 + 3$

$p(0) = 5(0) - 4(0)^2 + 3$

$= 0 - 4(0) + 3$

$= 0 - 0 + 3$

$\therefore\ p(0) = 3$

Hence, the value of the polynomial $p(x)$ at $x = 0$ is 3

(ii) $x = -1$

➤ $p(x) = 5x - 4x^2 + 3$

$p(-1) = 5(-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3$

$= -9 + 3$

$= -6$

Remember :
If the base is negative and the power is even number then its value is positive
Ex. $4(-1)^2$
$\therefore\ 4(1) = 4$

Hence, the value of the polynomial $p(x)$ at $x = -1$ is -6.

(iii) $x = 2$

➤ $p(x) = 5x - 4x^2 + 3$

$p(2) = 5(2) - 4(2)^2 + 3$

$= 10 - 4(4) + 3$

$= 10 - 16 + 3$

$= 13 - 16$

$= -3$

Hence, the value of the polynomial $p(x)$ at $x = 2$ is -3.

2) Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials :

(i) $p(y) = y^2 - y + 1$

➤ $p(0) = (0)^2 - (0) + 1$

$= 0 - 0 + 1 = 1$

$p(1) = (1)^2 - (1) + 1$

$= 1 - 1 + 1 = 1$

$p(2) = (2)^2 - (2) + 1$

$= 4 - 2 + 1 = 2 + 1 = 3$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

➤ $p(0) = 2 + (0) + 2(0)^2 - (0)^3$

$= 2 + 0 + 2(0) - 0$

$= 2 + 0 + 0 - 0 = 2$

➤ $p(1) = 2 + t + 2t^2 - t^3$

$= 2 + 1 + 2(1) - (1)$

$= 2 + 1 + 2 - 1$

$= 5 - 1 = 4$

➤ $p(2) = 2 + t + 2t^2 - t^3$

$= 2 + 2 + 2(2)^2 - (2)^3$

$= 2 + 2 + 2(4) - 8$

$= 2 + 2 + 8 - 8$

$= 4 + 8 - 8 = 4$

(iii) $p(x) = x^3$

➤ $p(0) = (0)^3 = 0$

$p(1) = (1)^3 = 1$

$p(2) = (2)^3 = 8$ $[\because 2 \times 2 \times 2 = 8]$

(iv) $p(x) = (x - 1)(x + 1)$

➤ $p(x) = (x - 1)(x + 1)$

$p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1 \times 1 = -1$

$p(1) = (1 - 1)(1 + 1) = (0)(2) = 0 \times 2 = 0$

$p(2) = (2 - 1)(2 + 1) = (1)(3) = 1 \times 3 = 3$

3) Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1,\ x = -\frac{1}{3}$

➤ $p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$

Therefore, $x = -\frac{1}{3}$ is the zero of the polynomial $p(x)$.

(ii) $p(x) = 5x - \pi,\ x = \frac{4}{5}$

➤ $p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi$ $\left[\because 5 \times \frac{4}{5} = 1 \times 4 = 4\right]$

$\therefore\ p\left(\frac{4}{5}\right) \neq 0$

Hence, $x = \frac{4}{5}$ is not a zero of the polynomial.

(iii) $p(x) = x^2 - 1,\ x = 1, -1$

➤ $p(1) = (1)^2 - 1 = 1 - 1 = 0$

$p(1) = 0$

$p(-1) = (-1)^2 = 1 - 1 = 0$

Hence, $x = 1$ and $x = -1$ are both zero of the polynomial.

(iv) $p(x) = (x + 1)(x - 2),\ x = -1, 2$

➤ $p(-1) = (-1 + 1)(-1 - 2) = (0)(-3) = 0$

Hence, $x = -1$ is a zero of the polynomial $p(x)$.

$p(x) = (x + 1)(x - 2)$

$\therefore\ p(2) = (2 + 1)(2 - 2) = (3)(0) = 0$

Hence, $x = 2$ is a zero of the polynomial $p(x)$.

UID : P1-C2-S3-Q1.1

(v) $p(x) = x^2, x = 0$ #

➤ $p(0) = (0)^2 = 0$

$\therefore\ p(0) = 0$

Hence, $x = 0$ is a zero of the polynomial $p(x)$.

(vi) $p(x) = lx + m,\ x = -\frac{m}{l}$

➤ $\therefore\ p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m$

$= (-m) + m = 0 \quad \left[\because l \times -\frac{m}{l} = -m\right]$

$\therefore\ p\left(-\frac{m}{l}\right) = 0$

Hence, $x = -\frac{m}{l}$ is a zero of the polynomial $p(x)$.

(vii) $p(x) = 3x^2 - 1,\ x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

➤ $\therefore\ p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 1 - 1 = 0$

$\left[\because \left(-\frac{1}{\sqrt{3}}\right)^2 = -\frac{1}{\sqrt{3}} \times -\frac{1}{\sqrt{3}} = \frac{1}{3}\right]$

$\therefore\ p\left(-\frac{1}{\sqrt{3}}\right) = 0$

Hence, $x = -\frac{1}{\sqrt{3}}$ is a zero of the polynomial $p(x)$.

➤ $p(x) = 3x^2 - 1$

$\therefore\ p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1$

$= 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3$

$\therefore\ p\left(\frac{2}{\sqrt{3}}\right) \neq 0$

Hence, $x = \frac{2}{\sqrt{3}}$ is not a zero of the polynomial $p(x)$.

(viii) $p(x) = 2x + 1,\ x = \frac{1}{2}$

➤ $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$

$\therefore\ p\left(\frac{1}{2}\right) \neq 0$

Hence, $x = \frac{1}{2}$ is not a zero of the polynomial $p(x)$.

\# UID : P1-C2-S3-Q3.5

4) Find the zero of the polynomial in each of the following cases :

(i) $p(x) = x + 5$

➤ $p(x) = 0$

$\therefore\ x + 5 = 0$

$\therefore\ x = -5$ Hence, -5 is the zero of $x + 5$.

(ii) $p(x) = x - 5$

➤ $p(x) = 0$

$\therefore\ x - 5 = 0$

$\therefore\ x = 5$ Hence, x is the zero of $x - 5$.

(iii) $p(x) = 2x + 5$

➤ $p(x) = 0$

$2x + 5 = 0$

$\therefore\ 2x = -5$

$\therefore\ x = -\frac{5}{2}$

Hence, $-\frac{5}{2}$ is the zero of $2x + 5$.

(iv) $p(x) = 3x - 2$

➤ $p(x) = 0$

$\therefore\ 3x - 2 = 0$

$\therefore\ 3x = 2$

$\therefore\ x = \frac{2}{3}$ Hence, $\frac{2}{3}$ is the zero of $3x - 2$.

(v) $p(x) = 3x$

➤ $p(x) = 0$

$\therefore\ 3x = 0$

$\therefore\ x = \frac{0}{3}$

$\therefore\ x = 0$ Hence, 0 is the zero of $3x$.

(vi) $p(x) = ax,\ a \neq 0$

➤ $p(x) = 0$

$\therefore\ ax = 0$

$\therefore\ x = \frac{0}{a}$

$\therefore\ x = 0$ Hence, 0 is the zero of ax.

(vii) $p(x) = cx + d,\ c \neq 0,\ c, d$ are real numbers.

➤ $p(x) = 0$

$\therefore\ cx + d = 0$

$\therefore\ cx = -d$

$\therefore\ x = -\frac{d}{c}$ Hence, $-\frac{d}{c}$ is the zero of $p(x)$.

Practice Work S12

2.1) Find the value of the polynomial.
$5x^3 - 2x^2 + 3x - 2$ at

(i) $x = 1$ # (ii) $x = 0$ (iii) $x = -1$

2.2) Find the value of each of the following polynomials at the indicated value of variable.

(i) $p(x) = x^3 - 3x^2 - 2x + 6$, $x = \sqrt{2}$
(ii) $p(x) = x^2 - 3x + 2$, $x = 2$
(iii) $p(y) = 2y^2 - 3y + 4$, $y = 0$
(iv) $p(t) = 2t^3 + t^2 - t - 1$, $t = (-2)$
(v) $p(t) = 4t^4 + 5t^3 - t^2 + 6$, $t = a$

2.3) Varify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = x^3 - 5x^2 + 6x$, $x = 0$
(ii) $p(x) = x^3 - 5x^2 + 6x$, $x = 1$
(iii) $p(x) = x^3 + 2x^2 - 5x - 6$, $x = -1$
(iv) $p(x) = ax + b$, $a \neq 0$

2.4) Find the zero of the polynomial in each of the following cases.

(i) $p(x) = x - 15$
(ii) $p(x) = 3x + 5$
(iii) $p(x) = 2x$
(iv) $p(x) = x^2 - 2x$
(v) $p(x) = 5x - 3$
(vi) $p(x) = 3$
(vii) $p(t) = 7t - 9$

2.5) Find $p(0)$, $p(1)$ and $p(2)$ for the each polynomial given follow.

(i) $p(x) = 5x - 4x^2 + 3$
(ii) $p(x) = x^2$
(iii) $p(x) = 5x^3 - 2x^2 + 3x - 2$

2.6) Check whether 2 and 0 are zeroes of the polynomial $x^2 - 2x$.

Answers

2.1) **(i)** 4 **(ii)** –2 **(iii)** –12

2.2) **(i)** 0 **(ii)** 0 **(iii)** 4 **(iv)** –11 **(v)** $4a^4 + 5a^3 - a^2 + 6$

2.3) **(i)** Yes **(ii)** No **(iii)** Yes **(iv)** Yes

2.4) **(i)** 15 **(ii)** $\frac{3}{5}$ **(iii)** 0 **(iv)** 0 **(v)** $\frac{3}{5}$ **(vi)** 3 is a constant polynomial. So it has no zero **(vii)** $\frac{9}{7}$

2.5) **(i)** $p(0) = 3$, $p(1) = 4$, $p(2) = -3$ **(ii)** $p(0) = 0$, $p(1) = 1$, $p(2) = 4$ **(iii)** $p(0) = -2$, $p(1) = 4$, $p(2) = 36$

2.6) $p(2) = 0$, $p(0) = 0$

2.3 - Things To Remember S1

1. Remainder Theorem :

1.1) Divide 15 by 6

$$\begin{array}{r|l} & 2 \text{ Quotient} \\ \text{Divisor } 6 & 15 \text{ Dividend} \\ & \underline{12} \\ & 3 \text{ Remainder} \end{array}$$

$15 = (6 \times 2) + 3$

$\therefore$ Dividend = (Divisor × Quotient) + Remainder

➤ Here, the remainder is 3 so 6 is not a factor of 15.

1.2) Divide 12 by 6

$$\begin{array}{r|l} & 2 \text{ Quotient} \\ \text{Divisior } 6 & 12 \text{ Dividend} \\ & \underline{12} \\ & 0 \text{ Remainder} \end{array}$$

$12 = (6 \times 2) + 0$

Dividend = (Divisor × Quotient) + Remainder

➤ Here, the remainder is 0. So 6 is a factor of 12.

1.3) Divide $p(x)$ by $g(x)$. Where $p(x) = x + 3x^2 - 1$ and $g(x) = 1 + x$.

➤ We write the dividend $x + 3x^2 - 1$ and the divisor $1 + x$ in the standard form.

➥ Dividend is $3x^2 + x - 1$ and divisor is $x + 1$.

$$\begin{array}{r|l} & 3x - 2 \quad \text{Quotient } q(x) \\ \text{Divisor } \begin{matrix} x+1 \\ g(x) \end{matrix} & 3x^2 + x - 1 \quad \text{Dividend } p(x) \\ & \underline{\overset{-}{}3x^2 \underset{-}{+} 3x} \\ & -2x - 1 \\ & -2x - 2 \\ & \underline{+ \quad +} \\ & 1 \quad \text{Remainder } r(x) \end{array}$$

Dividend = (Divisor × Quotient) + Remainder

$p(x) = g(x) \cdot q(x) + r(x)$

$\therefore \quad 3x^2 + x - 1 = (x + 1)(3x - 2) + 1$

➤ Here, the remainder is 1. so $x + 1$ is not a factor of $3x^2 + x - 1$.

UID : P1-C2-S12-Q2.1.1♦

1.4) Find the remainder obtained on dividing $p(x) = x^3 + 1$ by $x + 1$. #

➤ Standard form of $x^3 + 1$ is $x^3 + 0x^2 + 0x + 1$.

Standard form of $x + 1$ is $x + 1$

$$\begin{array}{r|l} & x^3 - x + 1 \\ \hline x+1 & x^3 + 0x^2 + 0x + 1 \\ & \underline{x^3 \pm x^2} \\ & -x^2 + 0x \\ & -x^2 - x \\ & \underline{+ \quad +} \\ & x + 1 \\ & x + 1 \\ & \underline{- \quad -} \\ & 0 \end{array}$$

Dividend = (Divisor × Quotient) + Remainder

$p(x) = g(x) \cdot q(x) + r(x)$

➤ Here, the remainder is 0.

$\therefore$ $x + 1$ is a factor of $x^3 + 1$.

➤ Here, $p(x) = x^3 + 1$ and the root of $x + 1 = 0$ is $x = -1$

$p(x) = x^3 + 1$

$p(-1) = (-1)^3 + 1 = -1 + 1 = 0$

Which is equal to the remainder obtained by actual division.

➤ **Remainder Theorem :**

Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If $p(x)$ is divided by the linear polynomial $x - a$, then the remainder is $p(a)$.

1.5) Factor Theorem :

➤ If $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number, then.

(i) $(x - a)$ is a factor of $p(x)$ if $p(a) = 0$ and

(ii) $p(a) = 0$, if $(x - a)$ is a factor of $p(x)$

1.6) Proof of factor Theorem :

➤ Let $p(x)$ is a polynomial of degree $n \geq 1$.

Let $p(x)$ is divided by $(x - a)$ the quotient is $q(x)$ and remainder is $r(x)$.

$\therefore \quad p(x) = (x - a)\ q(x) + r(x)$

➤ The degree of $x - a$ is 1. So the degree of $r(x)$ is 0.

$\therefore \quad r(x)$ is a constant or $r(x) = 0$

$p(x) = (x - a) \cdot q(x) + r$

$\therefore \quad p(a) = (a - a) \cdot q(a) + r$

$\therefore \quad p(a) = r$

➤ The remainder is $p(a)$.

➤ When the polynomial $p(x)$ is divided by $(x - a)$ the remainder is $p(a)$.

1.7) When $x^4 + x^3 - 2x^2 + x + 1$ is divided by $x - 1$ then find the remainder using remainder theorem.

➤ The root of $x - 1 = 0$ is $x = 1$.

$p(x) = x^4 + x^3 - 2x^2 + x + 1$

$\therefore \quad p(1) = (1)^4 + (1)^3 - 2(1)^2 + 1 + 1$

$= 1 + 1 - 2(1) + 1 + 1$

$= 1 + 1 - 2 + 1 + 1$

$\therefore \quad p(1) = 4 - 2$

$\therefore \quad p(1) = 2$

➤ By remainder theorem, when $x^4 + x^3 - 2x^2 + x + 1$ is divided by $x - 1$, the remainder is 2.

1.8) Remember :

(i) If $p(3) = 0$ then $x - 3$ is a factor of $p(x)$.

(ii) If $p(-2) = 0$ then $x + 2$ is a factor of $p(x)$.

(iii) If $p(x)$ is divided by $(x - 1)$ then $p(1) = 0$

(iv) If $p(x)$ is divided by $(x + 2)$ then $p(-2) = 0$

1.9) Find the value of k, if $x + 2$ is a factor of the polynomial $x^3 + 6x^2 + 4x + k$ OR Find the value of K when $x^3 + 6x^2 + 4x + k$ is divided by $x + 2$.

➤ $x + 2 = 0$

$\therefore\ x = -2$

$\therefore\ p(-2) = 0$

➤ $p(x) = x^3 + 6x^2 + 4x + k$

$p(-2) = (-2)^3 + 6(-2)^2 + 4(-2) + k$

$0 = -8 + 6(4) + 4(-2) + k$

$0 = -8 + 24 - 8 + k$

$0 = 24 - 16 + k$

$0 = 8 + k$

$8 + k = 0$

$\therefore \quad k = -8$

UID : P1-C2-S1-Q1.4

Exercise - 2.3

S4

1) Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by :

(i) $x + 1$ #

➤ There root of $x + 1 = 0$ is -1 [$\because$ $x + 1 = 0$, $x = -1$]

Taking, $x = -1$, we get

$p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$= (-1) + 3(1)^2 + 3(-1) + 1$

$= -1 + 3 - 3 + 1$

$\therefore$ $p(-1) = 0$

Hence, the remainder is 0.

(ii) $x - \frac{1}{2}$

➤ The root of $x - \frac{1}{2} = 0$ is $\frac{1}{2}$ $\left[\because x - \frac{1}{2} = 0 \therefore x = \frac{1}{2}\right]$

Taking, $x = \frac{1}{2}$, we get

$p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$

$= \frac{1}{8} + 3\left(\frac{1}{4}\right) + \frac{3}{2} + 1$

$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + \frac{1}{1}$

$= \frac{1 + 6 + 12 + 8}{8}$

$\therefore$ $p\left(\frac{1}{2}\right) = \frac{27}{8}$

Hence, the remainder is $\frac{27}{8}$.

(iii) x

➤ The root of $x = 0$ is 0 [$\because$ $x = 0$]

Taking, $x = 0$ we get

$p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p(0) = (0)^3 + 3(0)^2 + 3(0) + 1$

$= 0 + 3(0) + 3(0) + 1$

$\therefore$ $p(0) = 0 + 0 + 0 + 1 = 1$

Hence, the remainder is 1.

(iv) $x + \pi$

➤ The root of $x + \pi = 0$ is $-\pi$ [$\because$ $x + \pi = 0$ $\therefore$ $x = -\pi$]

Taking, $x = -\pi$ we get,

$p(x) = x^3 + 3x^2 + 3x + 1$

$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$= (-\pi)^3 + 3(\pi^2) + 3(-\pi) + 1$

Thus, the remainder is $-\pi^3 + 3\pi^2 - 3\pi + 1$

UID : P1-C2-S4-Q1.1

(v) $5 + 2x$

➤ Let $5 + 2x = 0$

$\therefore$ $2x = -5$

$\therefore$ $x = \frac{-5}{2}$

$\therefore$ The root of $5 + 2x = 0$ is $\frac{-5}{2}$

Taking, $x = \frac{-5}{2}$

$p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^3 + 3\left(-\frac{5}{2}\right)^2 + 3\left(-\frac{5}{2}\right) + 1$

$= -\frac{125}{8} + 3\left(\frac{25}{4}\right) + \left(-\frac{15}{2}\right) + 1$

$= -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + \frac{1}{1}$

$= \frac{-125 + 150 - 60 + 8}{8}$

$= \frac{-185 + 158}{8} = -\frac{27}{8}$

Hence, the remainder is $-\frac{27}{8}$.

2) Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

➤ The root of $x - a = 0$ is a.

$p(x) = x^3 - ax^2 + 6x - a$

$\therefore$ $p(a) = (a)^3 - a(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a$

$\therefore$ $p(a) = 5a$

➤ Thus, the remainder is $5a$.

3) Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

➤ Let $7 + 3x = 0$ $\therefore$ $3x = -7$ $\therefore$ $x = \frac{-7}{3}$

$\therefore$ The root of $7 + 3x = 0$ is $\frac{-7}{3}$.

$p(x) = 3x^3 + 7x$

Taking, $x = -\frac{7}{3}$ we get,

$p\left(-\frac{7}{3}\right) = 3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right)$

$= \frac{-343}{9} - \frac{49}{3}$

$= \frac{-490}{9}$

$= -\frac{490}{9} \neq 0$

$\therefore$ $7 + 3x$ is not a factor of $3x^3 + 7x$.

Here the remainder is not zero. So $7 + 3x$ is not a factor of $3x^3 + 7x$.

Practice Work S12

3.1) Find the remainder when the polynomial $x^3 + x^2 + x + 1$ is divided by

(i) $x + 1$ # (ii) $x - \frac{1}{2}$

(iii) $x + \pi$ (iv) x (v) $2x + 1$

3.2) If $x - 2$ is a factor of $x^2 + 3ax - 2a$ then find a.

3.3) Check whether $x - 2$ is a factor of $2x^3 - 13x^2 + 17x + 12$.

3.4) Find the remainder when $x^3 - 6x^2 + 5x + 5$ is divided by $x - 5$.

Answers

3.1) **(i)** 0 **(ii)** $\frac{15}{8}$ **(iii)** $-\pi^3 + \pi^2 - \pi + 1$ **(iv)** 1 **(v)** $\frac{5}{8}$ **3.2)** $a = -1$

3.3) The remainder is 10. So $x - 2$ is not a factor of $2x^3 - 13x^2 + 17x + 12$. **3.4)** 5

2.4 - Things To Remember S1

❖ **Factorisation of Polynomials.**

➤ **Splitting the middle term :**

➤ $ax^2 + bx + c$ is a quadratic polynomial $a, b, c \in$ R $a \neq 0$.

➤ Let $x^2 + lx + m$ is a quadratic polynomial.

comparing it with $ax^2 + bx + c$,

we get $a = 1$, $b = l$, $c = m$.

➤ Divide the co-efficient of x i.e. b in such a way that $l + m = b$ and $l \times m = ac$.

➤ $x^2 + lx + m = x^2 + (l)x + (m)$

$= x^2 + (a + b)x + ab$

$= x^2 + ax + bx + ab$

$= x(x + a) + b(x + a)$

$\therefore$ $x^2 + lx + m = (x + a)(x + b)$

Ex. factorise : $12x^2 - 7x + 1$

comparing $12x^2 - 7x + 1$ with

$ax^2 + bx + c$, we get,

$\therefore$ $a = 12$, $b = -7$, $c = 1$.

$\therefore$ $l + m = -7$ and $l \times m = 12$

$\therefore$ $l + m = (-4) + (-3)$ and $= l \times m$ $(-4)(-3)$

$\therefore$ $12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (3x - 1)(4x - 1)$

Thus, $12x^2 - 7x + 1 = (3x - 1)(4x - 1)$

Ex. Factorise $2x^2 + 7x + 3$

comparing $2x^2 + 7x + 3$ with

$ax^2 + bx + c$, we get,

$a = 2$, $b = 7$ and $c = 3$

$l + m = b$

$\therefore$ $l + m = 7$

$\therefore$ $(1 + 6) = 7$

$lm = ac$

$lm = 2 \times 3 = 6 = 1 \times 6$

$\therefore$ $2x^2 + 7x + 3 = 2x^2 + (1 + 6)x + 3$

$= 2x^2 + x + 6x + 3$

$\therefore$ $2x^2 + 7x + 3 = 2x^2 + x + 6x + 3$

$= x(2x + 1) + 3(2x + 1)$

$\therefore$ $2x^2 + 7x + 3 = (2x + 1)(x + 3)$

Exercise - 2.4 S5

1) Determine which of the following polynomials has $(x + 1)$ a factor :

(i) $x^3 + x^2 + x + 1$

➤ The root of $x + 1 = 0$ is $x = -1$

➤ If $x + 1$ is a factor of $p(x) = x^3 + x^2 + x + 1$ then $p(-1) = 0$

$p(x) = x^3 + x^2 + x + 1$

$\therefore$ $p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$= 0$

$\therefore$ $x + 1$ is a factor of $p(x) = x^3 + x^2 + x + 1$.

UID : P1-C2-S12-Q3.1.1♦

(ii) $x^4 + x^3 + x^2 + x + 1$ #

➤ $p(x) = x^4 + x^3 + x^2 + x + 1$

The root of $x + 1 = 0$ is $x = -1$.

$p(x) = x^4 + x^3 + x^2 + x + 1$

$\therefore\ p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1$

$= 1 \neq 0$

Here, the remainder is not zero.

So, $x + 1$ is not a factor of $p(x) = x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

➤ $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

The root of $x + 1 = 0$ is $x = -1$

$p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$\therefore\ p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1$

$= 1 \neq 0$

Here the remainder is not zero.

So, $x + 1$ is not a factor of $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$.

(iv) $x^3 - x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$

➤ $p(x) = x^3 - x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$

The root of $x + 1 = 0$ is $x = -1$

$\therefore\ p(-1) = (-1)^3 - (-1)^2 - \left(2+\sqrt{2}\right)(-1) + \sqrt{2}$

$= -2 + 2 + \sqrt{2} + \sqrt{2}$

$= 2\sqrt{2} \neq 0$

Here, the remainder is not zero

So, $x + 1$ is not a factor of

$p(x) = x^3 - x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$.

2) Use the factor theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = 2x^3 + x^2 - 2x - 1,\ g(x) = x + 1$

➤ $x + 1 = 0 \therefore x = -1$

$p(x) = 2x^3 + x^2 - 2x - 1$

$\therefore\ p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= 2(-1) + (1) + 2 - 1$

$= -2 + 1 + 2 - 1$

$\therefore\ p(-1) = 0$

Yes, $g(x)$ is a factor of $p(x)$.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1,\ g(x) = x + 2$

➤ $x + 2 = 0 \therefore x = -2$

$p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore\ p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= (-8) + 3(4) + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -14 + 13$

$\therefore\ p(-2) = -1 \neq 0$

No, $g(x)$ is not a factor of $p(x)$.

(iii) $p(x) = x^3 - 4x^2 + 4x + 6,\ g(x) = x - 3$

➤ $x - 3 = 0 \therefore x = 3$

$p(x) = x^3 - 4x^2 + x + 6$

$\therefore\ p(3) = (3)^3 - 4(3)^2 + (3) + 6$

$= 27 - 36 + 3 + 6$

$= 36 - 36 = 0$

$\therefore\ p(3) = 0$

Yes, $g(x)$ is a factor of $p(x)$.

3) Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = x^2 + x + k$

➤ $(x - 1)$ is a factor of $p(x)$.

So, $p(1) = 0$.

$p(x) = x^2 + x + k$

$\therefore\ p(1) = (1)^2 + (1) + k$

$0 = 1 + 1 + k$

$0 = 2 + k$

$\therefore\ k = -2$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

➤ $x - 1$ is a factor of $p(x)$.

So, $p(1) = 0$

$p(x) = 2x^2 + kx + \sqrt{2}$

$\therefore\ p(1) = 2(1)^2 + k(1) + \sqrt{2}$

$\therefore\ 0 = 2(1) + k + \sqrt{2}$

$\therefore\ 0 = 2 + k + \sqrt{2}$

$\therefore\ -2 - \sqrt{2} = k$

$\therefore\ -\left(2+\sqrt{2}\right) = k$

$\therefore\ k = -\left(2+\sqrt{2}\right)$

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(iii) $p(x) = kx^2 - \sqrt{2}x + 1$ #

➤ $x - 1$ is a factor of $p(x)$.

So, $p(1) = 0$.

$p(x) = kx^2 - \sqrt{2}x + 1$

$\therefore\ p(1) = k(1)^2 - \sqrt{2}(1) + 1$

$\therefore\ 0 = k(1) - \sqrt{2} + 1$

$\therefore\ 0 = k - \sqrt{2} + 1$

$\therefore\ \sqrt{2} - 1 = k$

$\therefore\ k = \sqrt{2} - 1$

(iv) $p(x) = kx^2 - 3x + k$

➤ $x - 1$ is a factor of $p(x)$

So, $p(1) = 0$.

$p(x) = kx^2 - 3x + k$

$\therefore\ p(1) = k(1)^2 - 3(1) + k$

$\therefore\ 0 = k - 3 + k$

$\therefore\ 0 = 2k - 3$

$\therefore\ 3 = 2k$

$\therefore\ \frac{3}{2} = k$

$\therefore\ k = \frac{3}{2}$

4) Factorise:

(i) $12x^2 - 7x + 1$

➤ Here,

Comparing $12x^2 - 7x + 1$ with

$ax^2 + bx + c$ we get,

$a = 12,\ b = -7,\ c = 1$

Now, $l + m = b = -7$,

$l \times m = ac = 12$

$l = -4,\ m = -3 \Rightarrow l + m = 4 - 3 = -7$

$l \times m = (-4)\ (-3) = 12$

Now $12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$

$= 4x\ (3x - 1) - 1(3x - 1)$

$= (3x - 1)\ (4x - 1)$

Hence, $12x^2 - 7x + 1 = (3x - 1)\ (4x - 1)$

(ii) $2x^2 + 7x + 3$

➤ Comparing $2x^2 + 7x + 3$ with

$ax^2 + bx + c$, we get, $a = 2$, $b = 7$ and $c = 3$

Now, $l + m = b = 7$ and $lm = ac = 6$

$l = 1$ and $m = 6$

Now, $2x^2 + 7x + 3 = 2x^2 + x + 6x + 3$

$= x\ (2x + 1) + 3(2x + 1)$

$= (2x + 1)\ (x + 3)$

Thus, $2x^2 + 7x + 3 = (2x + 1)\ (x + 3)$

(iii) $6x^2 + 5x - 6$

➤ Comparing $6x^2 + 5x - 6$ with

$ax^2 + bx + c$, we get, $a = 6$, $b = 5$ and $c = -5$

Now, $l + m = b = 5$ and $l \times m = ac = 36$

$\therefore\ l + m = 9 + (-4) = 5$

$\therefore\ l \times m = (9)\ (-4) = -36$

$\therefore\ l = 9,\ m = -4$

Now, $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x\ (2x + 3) - 2(2x + 3)$

$= (2x + 3)\ (3x - 2)$

Thus, $6x^2 + 5x - 6 = (2x + 3)\ (3x - 2)$

(iv) $3x^2 - x - 4$

➤ Comparing $3x^2 - x - 4$

With $ax^2 + bx + c$ we get,

$a = 3$, $b = -1$ and $c = -4$

Now, $l + m = b = -1$ and $l \times m = ac = -12$

$\therefore\ l + m = -4 + 3 = -1$

$l \times m = (-4)\ (3) = -12$

$\therefore\ 3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x\ (3x - 4) + 1(3x - 4)$

$= (3x - 4)\ (x + 1)$

Thus, $3x^2 - x - 4 = (3x - 4)\ (x + 1)$

5) Factorise:

(i) $x^3 - 2x^2 - x + 2$

➤ $x^3 - 2x^2 - x + 2 = x^3 - 2x^2 - x + 2$

$= x^3 - x - 2x^2 + 2$

$= x\ (x^2 - 1) - 2(x^2 - 1)$

$= (x^2 - 1)\ (x - 2)$

$= (x - 1)\ (x + 1)\ (x - 2)$

Thus, $x^3 - 2x^2 - x + 2 = (x - 1)\ (x + 1)\ (x - 2)$

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(ii) $x^3 - 3x^2 - 9x - 5$ #

➤ We know that $p(1) = 0$ or $p(-1) = 0$.

$p(x) = x^3 - 3x^2 - 9x - 5$

$\therefore\ p(1) = (1)^3 - 3(1)^2 - 9(1) - 5$

$= 1 - 3 - 9 - 5$

$= -16 \neq 0$

$\therefore\ p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5$

$= 0$

Here, $p(1) \neq 0$ but $p(-1) = 0$

$\therefore$ By factor theorem, $x - (-1)$ is a factor of $p(x)$.

i. e. $x + 1$ is a factor of $p(x)$

$$\frac{x^3 - 3x^2 - 9x - 5}{x + 1} = x^2 - 4x - 5$$

$$\begin{array}{r|l} & x^2 - 4x - 5 \\ \hline x + 1 & x^3 - 3x^2 - 9x - 5 \\ & \underline{\overset{-}{x^3} \overset{-}{+ x^2}} \\ & -4x^2 - 9x \\ & \underline{\overset{+}{-4x^2} \overset{+}{-4x}} \\ & -5x - 5 \\ & \underline{\overset{+}{-5x} \overset{+}{-5}} \\ & 0 \end{array}$$

$\therefore\ x^3 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5)$

$= (x + 1)[x^2 - 5x + x - 5]$

$= (x + 1)[x(x - 5) + 1\ (x - 5)]$

$= (x + 1)[(x - 5)(x + 1)]$

$\therefore\ x^3 - 3x^2 - 9x - 5 = (x + 1)(x - 5)(x + 1)$

(iii) $x^3 + 13x^2 + 32x + 20$

➤ $p(x) = x^3 + 13x^2 + 32x + 20$

$\therefore\ p(1) = (1)^3 + 13(1)^2 + 32(1) + 20$

$= 1 + 13 + 32 + 20$

$= 66 \neq 0$

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

$= 0$

Here, $p(1) \neq 0$ but $p(-1) = 0$

So, by factor theorem $x - (-1)$ is a factor of $p(x)$.

i.e. $x + 1$ is a factor of $p(x)$.

$$\frac{x^3 + 13x^2 + 32x + 20}{x + 1} = x^2 + 12x + 20$$

$$\begin{array}{r|l} & x^2 + 12x + 20 \\ \hline x + 1 & x^3 + 13x^2 + 32x + 20 \\ & \underline{\overset{-}{x^3} \overset{-}{+ x^2}} \\ & 12x^2 + 32x \\ & \underline{\overset{-}{12x^2} \overset{-}{+ 12x}} \\ & 20x + 20 \\ & \underline{\overset{-}{20x} \overset{-}{+ 20}} \\ & 0 \end{array}$$

$\therefore\ x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20)$

$= (x + 1)[x^2 + 2x + 10x + 20]$

$= (x + 1)[x(x + 2) + 10\ (x + 2)]$

$= (x + 1)(x + 2)(x + 10)$

(iv) $2y^3 + y^2 - 2y - 1$

➤ We know that $p(1) = 0$ or $p(-1) = 0$.

$p(y) = 2y^3 + y^2 - 2y - 1$

$\therefore\ p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$= 2(1) + (1) - 2 - 1$

$= 2 + 1 - 2 - 1$

$= 0$

$\therefore\ p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1$

$= 0$

$\therefore$ By factor theorem, $y - 1$ is a factor of $p(y)$. Also $y - (-1)$ is a factor of $p(y)$.

i.e. $y + 1$ is a factor of $p(y)$.

$$\frac{2y^3 + y^2 - 2y - 1}{y - 1} = 2y^2 + 3y + 1$$

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$$\begin{array}{r|l} & 2y^2+3y+1 \\ \hline y-1 & 2y^3+y^2-2y-1 \\ & \underline{2y^3-2y^2} \\ & \quad 3y^2-2y-1 \\ & \quad \underline{3y^2-3y} \\ & \qquad y-1 \\ & \qquad \underline{y-1} \\ & \qquad\quad 0 \end{array}$$

$\therefore\ 2y^3+y^2-2y-1 = (y-1)\ (2y^2+3y+1)$

$= (y-1)\ [2y^2+2y+y+1)]$

$= (y-1)\ [2y\,(y+1)+1\ \ (y+1)]$

$2y^3+y^2-2y-1 = (y-1)\ (y+1)(2y+1)$

$$\frac{2y^3+y^2-2y-1}{y+1} = 2y^2-y-1$$

$$\begin{array}{r|l} & 2y^2-y-1 \\ \hline y+1 & 2y^3+y^2-2y-1 \\ & \underline{2y^3+2y^2} \\ & \quad -y^2-2y \\ & \quad \underline{-y^2-y} \\ & \qquad -y-1 \\ & \qquad \underline{-y-1} \\ & \qquad\quad 0 \end{array}$$

$\therefore\ 2y^3+y^2-2y-1 = (y+1)\ (2y^2-y-1)$

$= (y+1)\ [2y^2-2y+y-1)]$

$= (y+1)\ [2y\,(y-1)+1\ \ (y-1)]$

$2y^3+y^2-2y-1 = (y+1)\ (y-1)(2y+1)$

Practice Work S12

4.1) Determine which of the following polynomials has $(x+1)$ a factor :

(i) x^2+6x+7 #

(ii) $x^3+10x^2+23x+14$

(iii) $3x^2+7x+4$

(iv) $21x^3+16x^2+4x+9$

4.2) Determine which of the following polynomials has $(x-1)$ a factor :

(i) $2x^3-3x^2+3x-2$

(ii) $4x^3+x^4-x+1$

(iii) $5x^4-4x^3-2x+1$

(iv) $3x^3+x^2+x+11$

4.3) Use the factor theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = x^3+10x^2+23x+14,\ g(x) = x+1$

(ii) $p(x) = x^3+4x^2+4x+1,\ g(x) = x+1$

(iii) $p(x) = x^3+2x^2-5x-6,\ g(x) = x-1$

4.4) Find the value of k, if $g(x)$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = kx^3+3x^2+7x+13,\ g(x) = x+3$

(ii) $p(x) = kx^4-7x^3-3x^2-2x-8,\ g(x) = x-4$

(iii) $p(x) = x^2+(4-k)x+2,\ g(x) = x-2$

4.5) Factorise :

(i) $3x^2+7x+4$

(ii) $15x^2+16x+4$

(iii) $-21x^2+16x+5$

(iv) $x^2-19x+84$

4.6) Factorise :

(i) x^3-3x^2-6x+8

(ii) x^3+2x^2-5x-6

(iii) x^3-2x^2-5x+6

(iv) $2x^3-13x^2+23x-12$

Answers

4.1) **(i)** No **(ii)** Yes **(iii)** Yes **(iv)** Yes

4.2) **(i)** Yes **(ii)** No **(iii)** Yes **(iv)** No

4.3) **(i)** Yes **(ii)** Yes **(iii)** No

4.4) **(i)** $(k = 1)$ **(ii)** $(k = 2)$ **(iii)** $(k = 7)$

4.5) **(i)** $(x+1)\ (3x+4)$ **(ii)** $(3x+2)\ (5x+2)$ **(iii)** $-(x-1)\ (21x+5)$ **(iv)** $(x-7)\ (x-12)$

4.6) **(i)** $(x-1)\ (x-4)\ (x+2)$ **(ii)** $(x+1)\ (x+3)\ (x-2)$ **(iii)** $(x-1)\ (x+2)\ (x-3)$ **(iv)** $(x-1)\ (2x-3)\ (x-4)$

UID : P1-C2-S12-Q4.1.1♦

2.5 - Things To Remember S1

2. Algebraic Identities :

2.1) What is algebraic identities ? #

An algebraic identities is an algebraic expression true for all values of the variables occurring in it.

Identity No.	
(i)	$(x + y)^2 = x^2 + 2xy + y^2$
(ii)	$(x - y)^2 = x^2 - 2xy + y^2$
(iii)	$x^2 - y^2 = (x - y)(x + y)$
(iv)	$(x + a)(x + b) = x + (a + b)x + ab$
(v)	$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
(vi)	$(x + y)^3 = x^3 + y^3 + 3xy(x + y) = x^3 + y^3 + 3x^2y + 3xy^2$
(vii)	$(x - y)^3 = x^3 - y^3 - 3xy(x - y) = x^3 - y^3 - 3x^2y + 3xy^2$
(viii)	$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Exercise - 2.5 S6

1) Use suitable identities to find the following products :

(i) $(x + 4)(x + 10)$

➤ $(x + a)(x + b) = x^2 + (a + b)x + ab$

$(x + 4)(x + 10) = x^2 + (4 + 10)x + (4 \times 10)$

(where $a = 4$, $b = 10$)

$= x^2 + 14x + 40$

(ii) $(x + 8)(x - 10)$

➤ $(x + a)(x + b) = x^2 + (a + b)x + ab$

$(x + 8)(x - 10) = x^2 [8 + (-10)] x + [8 \times (-10)]$

(where $a = 8$, $b = -10$)

$= x^2 + [-2]x + [-80]$

$= x^2 - 2x - 80$

(iii) $(3x + 4)(3x - 5)$

➤ $(x + a)(x + b) = x^2 + (a + b)x + (ab)$

$(3x + 4)(3x - 5) = (3x)^2 + [4 + (-5)] 3x + [4(-5)]$

(where $a = 4$, $b = -5$, $x = 3x$)

$= 9x^2 + [-1] 3x + [-20]$

$= 9x^2 - 3x - 20$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

➤ $(a + b)(a - b) = a^2 - b^2$

$\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = [y^2]^2 - \left[\frac{3}{2}\right]^2$

$= y^4 - \frac{9}{4}$

(v) $(3 - 2x)(3 + 2x)$

➤ $(a + b)(a - b) = a^2 - b^2$

$(3 - 2x)(3 + 2x) = (3)^2 - (2x)^2$

$= 9 - 4x^2$

2) Evaluate the following products without multiplying directly :

(i) 103×107

➤ $103 \times 107 = (100 + 3)(100 + 7)$

$(x + a)(x + b) = x^2 + (a + b)x + (a \times b)$

$= (100)^2 + (3 + 7)100 + (3 \times 7)$

$= 10000 + (10)100 + 21$

$= 10000 + 1000 + 21$

$= 11021$

(ii) 95×96

➤ $95 \times 96 = (100 - 5)(100 - 4)$

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$= (100)^2 + [(-5) + (-4)] 100 + [(-5) \times (-4)]$

$= 1000 + [-9] 100 + [20]$

$= 10000 - 900 + 20$

$= 9120$

(iii) 104×96

➤ $104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - (4)^2$ [$\because (a + b)(a - b) = a^2 - b^2$]

$= 10000 - 16$

$= 9984$

3) Factorise the following using appropriate identities :

(i) $9x^2 + 6xy + y^2$

➤ $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x + y)^2$ [$\because a^2 + 2ab + b^2 = (a + b)^2$]

$= (3x + y)(3x + y)$

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(ii) $4y^2 - 4y + 1$ #

➤ $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)^2 \quad [\because a^2 - 2ab + b^2 = (a - b)^2]$

$= (2y - 1)(2y - 1)$

(iii) $x^2 - \frac{y^2}{100}$

➤ $x^2 - \frac{y^2}{100} = (x)^2 - \left(\frac{y}{10}\right)^2 = \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$

$[\because a^2 - b^2 = (a + b)(a - b)]$

4) Expand each of the following, using suitable identities :

(i) $(x + 2y + 4z)^2$

➤ $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(x + 2y + 4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(ii) $(2x - y + z)^2$

➤ $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(2x - y + 2)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2$

➤ $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(x)(y) + 2(y)(z) + 2(z)(x)$

$(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$

(iv) $(3a - 7b - c)^2$

➤ $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$

(v) $(-2x + 5y - 3z)^2$

➤ $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$

(vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

➤ $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2 = \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2(1)\left(\frac{1}{4}a\right)$

$= \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

5) Factorise :

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

➤ $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

➤ $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)(y) + 2(2\sqrt{2}z)(y) + 2(2\sqrt{2}z)(-\sqrt{2}x)$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

$= \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)\left(-\sqrt{2}x + y + 2\sqrt{2}z\right)$

[6] Write the following cubes in expanded form :

(i) $(2x + 1)^3$

➤ Identify (vi) : $(x + y)^3 = (x)^3 + (y)^3 + 3xy(x + y)$

$(2x + 1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1)$

$= 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a - 3b)^3$

➤ Identify (vii) : $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)$

$= 8a^3 - 27b^3 - 18ab(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $\left(\frac{3}{2}x + 1\right)^3$

➤ Identify (vi) : $(x + y)^3 = (x)^3 + (y)^3 + 3xy(x + y)$

$\left(\frac{3}{2}x + 1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left[\frac{3}{2}x + 1\right]$

$= \frac{27}{8}x^3 + 1 + \frac{9x}{2}\left[\frac{3}{2}x + 1\right]$

$= \frac{27x^3}{8} + 1 + \frac{27x^2}{4} + \frac{9x}{2}$

$= \frac{27x^3}{8} + \frac{27x^2}{4} + \frac{9x}{2} + 1$

$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

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(iv) $\left(x - \frac{2}{3}y\right)^3$ #

➤ Identify (vii) : $(x - y)^3 = x^3 - y^3 - 3xy\ (x - y)$

$\left(x - \frac{2}{3}y\right)^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x - \frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2xy\left(x - \frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - \left[(2xy)\ x - (2xy)\ \frac{2}{3}y\right]$

$= x^3 - \frac{8}{27}y^3 - \left[2x^2y - \frac{4}{3}xy^2\right]$

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

[7] Evaluate the following using suitable identities :

(i) $(99)^3$

➤ $99 = 100 - 1$

$(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3(100)\ (1)\ (100 - 1)$

$= 1000000 - 1 - 300(99)$

$= 1000000 - 1 - 29700$

$= 1000000 - 29701 = 970299$

(ii) $(102)^3$

➤ $102 = 100 + 2$

$(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)\ (2)\ (100 + 2)$

$= 1000000 + 8 + 600\ (100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

➤ $(998) = 1000 - 2$

$(998)^3 = (1000)^3 - (2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)\ [1000 - 2]$

$= 1000000000 - 8 - 6000\ (1000 - 2)$

$= 1000000000 - 8 - 6000000 + 12000$

$= 994011992$

8) Factorise each of the following :

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

➤ $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 6ab\ (2a + b)$

$= (2a)^3 + (b)^3 + 3\ (2a)\ (b)\ (2a + b)$

$= (2a + b)^3$

$= (2a + b)\ (2a + b)\ (2a + b)$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

➤ $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - (b)^3 - 3(2a)\ (b)\ (2a - b)$

$= (2a - b)^3$

$= (2a - b)\ (2a - b)\ (2a - b)$

(iii) $27 - 125a^3 - 135a + 225a^2$

➤ $27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 3(3)(5a)\ (3 - 5a)$

$= (3 - 5a)^3$

$= (3 - 5a)\ (3 - 5a)\ (3 - 5a)$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

➤ $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3\ (4a)\ (3b)\ (4a - 3b)$

$= (4a - 3b)^3$

$= (4a - 3b)\ (4a - 3b)\ (4a - 3b)$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

➤ $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3\,(3p)\left(\frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

$= \left(3p - \frac{1}{6}\right)^3$

$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

9) Verify :

(i) $x^3 + y^3 = (x + y)\ (x^2 - xy + y^2)$

➤ R.H.S. $= (x + y)\ (x^2 - xy + y^2)$

$= x\,(x^2 - xy + y^2) + y\,(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3$

$= x^3 + y^3$

$\therefore$ R.H.S. = L.H.S.

(ii) $x^3 - y^3 = (x - y)\ (x^2 + xy + y^2)$

➤ R.H.S. $= (x - y)\ (x^2 + xy + y^2)$

$= x\ (x^2 + xy + y^2) - y\ (x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - yx^2 - xy^2 - y^3$

$= x^3 - y^3$

$\therefore$ R.H.S. = L.H.S.

10) Factorise each of the following :

(i) $27y^3 + 125z^3$ **(ii)** $64m^3 - 343n^3$

Remember : (i) $x^3 + y^3 = (x + y)\ (x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)\ (x^2 + xy + y^2)$

(i) $x^3 + y^3 = (x + y)\ (x^2 + y^2 - xy)$

➤ $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$= (3y + 5z)\ [(3y)^2 - (3y)\ (5z) + (5z)^2]$

$= (3y + 5z)\ [9y^2 - 15yz + 25z^2]$

$= (3y + 5z)\ [9y^2 + 25z^2 - 15yz]$

UID : P1-C2-S6-Q6.4

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ #

➤ $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

$= (4m - 7m)[(4m)^2 + (4m)(7n) + (7n)^2]$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

$= (4m - 7n)(16m^2 + 49n^2 + 28mn)$

11) Factorise : $27x^3 + y^3 + z^3 - 9xyz$

➤ **Remember :** $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

➤ $27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)[(3x)^2 + y^2 + z^2 - (3x \times y) - (yz) - (z \times 3x)]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

12) Verify that : $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

➤ R.H.S. $= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}(x + y + z)[(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (z^2 + x^2 - 2xz)]$

$= \frac{1}{2}(x + y + z)[x^2 + y^2 + y^2 + z^2 + z^2 + x^2 - 2xy - 2yz - 2zx]$

$= \frac{1}{2}(x + y + z)[2(x^2 + y^2 + z^2 - xy - yz - zx)]$

$= 2 \times \frac{1}{2} \times (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= x^3 + y^3 + z^3 - 3xyz$

$\therefore$ R.H.S. = L.H.S.

13) If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$

➤ $x + y + z = 0$

$\therefore x + y = -z$

$\therefore (x + y)^3 = (-z)^3$ ($\because$ Taking, cube both side)

$\therefore x^3 + y^3 + 3xy(x + y) = -z^3$

$\therefore x^3 + y^3 + z^3 - 3xyz = 0$

$\therefore x^3 + y^3 + z^3 = 3xyz$

Thus, $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$.

14) Without actually calculating the cubes, find the value of each of the following :

(i) $(-12)^3 + (7)^3 + (5)^3$

➤ Let $x = -12$, $y = 7$ and $z = 5$

$x + y + z = (-12) + (7) + (5) = 0$

We know that $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

$\therefore (-12)^3 + (7)^3 + (5)^3 = 3[(-12)(7)(5)]$

$= 3(-420) = -1260$

Thus, $(-12)^3 + (7)^3 + (5)^3 = -1260$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

➤ Let $x = 28$, $y = -15$ and $z = -13$

$\therefore x + y + z = (28) + (-15) + (-13) = 0$

We know that $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

$= 3(5460) = 16380$

Hence, $(28)^3 + (-15)^3 + (-13)^3 = 16380$

15) Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given :

(i) Area : $25a^2 - 35a + 12$

➤ Area of the rectangle = length × Breadth

Area $= 25a^2 - 35a + 12$

We factorise $25a^2 - 35a + 12$

The co-efficient of $a = -35 = (-20) + (-15)$

Also, $25 \times 12 = 300 = (-20) \times (-15)$

$\therefore 25a^2 - 35a + 12 = 25a^2 - 20a - 15a + 12$

$= 5a(5a - 4) - 3(5a - 4)$

$= (5a - 4)(5a - 3)$

Thus, the Length and breadth of the rectangle are $5a - 4$ and 5a – 3 respectively.

(ii) Area : $35y^2 + 13y - 12$

➤ Area of the rectangle = (2) Length breadth

Area $= 35y^2 + 13y - 12$

The co-efficient of $y = 13 = 28 + (-15)$

$35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (5y + 4)(7y - 3)$

Thus, the length and breadth of the rectangle are $5y + 4$ and $7y - 3$.

16) What are the possible expressions for the dimensions of the cuboids whose volumes are given below ?

(i) Volume : $3x^2 - 12x$

➤ Volume = Length × Breadth × Height

$= 3x^2 - 12x$

$= 3x(x - 4)$

$= 3 \times x \times x - 4$

$\therefore$ The dimension of cuboids are 3, x and $(x - 4)$.

UID : P1-C2-S6-Q10.2

(ii) Volume : $12ky^2 + 8ky - 20k$ #

➤ Volume = length × breadth × height

$= 12ky^2 + 8ky - 20k$

$= 4[k\ (3y^2 + 2y - 5)]$

$= 4 \times k \times (3y^2 + 2y - 5)$

$= 4k\ [3y^2 - 3y + 5y - 5]$

$= 4k\ [3y\ (y - 1) + 5\ (y - 1)]$

$= 4k\ (3y + 5)\ (y - 1)$

Hence, the dimension of the cuboid are $4k$, $3y + 5$, $y - 1$.

Practice Work S12

5.1) Use suitable identities to find the following products.

(i) $(x + 3)\ (x + 5)$

(ii) $(x - 7)\ (x - 12)$

(iii) $(5 - 4x)\ (7 - 4x)$

(iv) $(5 - 3x)\ (5 + 3x)$

(v) $\left(x + \frac{3}{2}\right)\left(2x + \frac{5}{3}\right)$

(vi) $\left(3x + \frac{3}{2}\right)\left(3x + \frac{5}{2}\right)$

5.2) Evaluate the following products without multiplying directly.

(i) 105×102

(ii) 97×103

(iii) 77×83

5.3) Factorise the following using appropriate identities.

(i) $25x^2 + 10x + 1$ (ii) $4y^2 - 12y + 9$

(iii) $25x^2 - \frac{y^2}{36}$

5.4) Factorise.

(i) $x^2 + 4y^2 + z^2 - 4xy - 4yz + 2yx$

(ii) $4a^2 + 9b^2 + c^2 + 12ab - 6bc - 4ca$

(iii) $4x^4 + 9y^6 + 25 - 12x^2y^3 + 30y^3 - 20x^2$

(iv) $a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$

(v) $4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

5.5) Expand each of the following using suitable identities.

(i) $\left(\frac{1}{5}a - \frac{1}{3}b + 1\right)^2$ (ii) $(3a - 5b - 7c)^2$

(iii) $(4a - 2b - 3c)^2$

5.6) Write the following cubes in expanded form.

(i) $(2x + 3y)^3$ (ii) $(2x - 1)^3$

(iii) $(3x - 2y)^3$ (iv) $\left(\frac{5}{2}x + 1\right)^3$

5.7) Evaluate the following using suitable identities.

(i) $(27)^3$ (ii) $(108)^3$ (iii) $(893)^3$

5.8) Factorise each of the following.

(i) $8x^3 + 27y^3 + 36x^2y + 54xy^2$

(ii) $8a^3 - 27b^3 - 36a^2b + 54\ ab^2$

(iii) $27x^3 - 64y^3 - 108x^2y + 144\ xy^2$

(iv) $8x^3 + y^3 + 12x^2y + 6xy^2$

(v) $64p^3 - \frac{1}{343} - \frac{48p^2}{7} + \frac{12p}{49}$

5.9) Verify :

(i) $x^3 + 27 = (x + 3)\ (x^2 - 3x + 9)$

(ii) $x^3 - 64 = (x - 4)\ (x^2 + 4x + 16)$

5.10) Factorise each of the following.

(i) $27x^3 - 64y^3$ (ii) $125y^3 - 8$

5.11) Factorise : $m^3 + n^3 + p^3 - 3mnp$

5.12) Verify : $a^3 + b^3 + c^3 - 3\ abc = \frac{1}{2}\ (a + b + c)\ [(a - b)^2 + (b - c)^2 + (c - a)^2]$

5.13) If $a + b + c = 0$ show that $a^3 + b^3 + c^3 = 3\ abc$

5.14) Without actually ckalculatting the cubes, find the value of each of the following.

(i) $(-7)^3 + (12)^3 + (-5)^3$ (ii) $(18)^3 + (15)^3 - (33)^3$

5.15) Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given.

(i) Area $49x^2 - 56x + 15$

(ii) Area $25y^2 + 35y + 6$

5.16) What are the possible expressions for the dimensions of the cuboid whose volumes are given below.

(i) volumes $3x^2 - 75x$

(ii) volumes $8k^2y + 4ky - 40k$

UID : P1-C2-S6-Q16.2

Answers

5.1) **(i)** $x^2 + 8x + 15$ **(ii)** $x^2 - 19x + 84$
(iii) $16x^2 - 48x + 35$ **(iv)** $25 - 9x^2$
(v) $\frac{1}{6}(12x^2 + 28x + 15)$ **(vi)** $\frac{1}{4}(36x^2 + 48x + 15)$

5.2) **(i)** 10710 **(ii)** 9991 **(iii)** 6391

5.3) **(i)** $(5x + 1)^2$ **(ii)** $(2y - 3)^2$ **(iii)** $\left(5x - \frac{y}{6}\right)\left(5x + \frac{y}{6}\right)$

5.4) **(i)** $(x - 2y + z)(x - 2y + z)$ **(ii)** $(2a + 3b - c)(2a + 3b - c)$ **(iii)** $(2x^2 - 3y^3 - 5)(2x^2 - 3y^3 - 5)$ **(iv)** $(a - b - c)(a - b - c)$ **OR** $(-a + b + c)(-a + b + c)$ **(v)** $(2x - y + z)(2x - y + z)$

5.5) **(i)** $\frac{1}{25}a^2 + \frac{1}{9}b^2 + 1 - \frac{2}{15}ab - \frac{2b}{3} + \frac{2a}{5}$ **(ii)** $9a^2 + 25b^2 + 49c^2 - 30ab + 70bc - 42ac$
(iii) $16a^2 + 4b^2 + ac^2 - 16ab + 12bc - 24ac$

5.6) **(i)** $8x^3 + 27y^3 + 36x^2y + 54xy^2$ **(ii)** $8x^3 - 1 - 12x^2 + 6x$ **(iii)** $27x^3 - 8y^3 - 54x^2y^2 + 36xy^2$
(iv) $\frac{125x^3}{8} + 1 + \frac{75x^2}{4} + \frac{15x}{2}$

5.7) **(i)** 19683 **(ii)** 12,59,712 **(iii)** 71,21,21,9571

5.8) **(i)** $(2x + 3y)(2x + 3y)(2x + 3y)$ **(ii)** $(2x - 3y)(2x - 3y)(2x - 3y)$ **(iii)** $(3x - 4y)(3x - 4y)(3x - 4y)$
(iv) $(2x + y)(2x + y)(2x + y)$ **(v)** $\left(4p - \frac{1}{7}\right)\left(4p - \frac{1}{7}\right)\left(4p - \frac{1}{7}\right)$

5.10) $(3x - 4y)(9x^2 + 12xy + 16y^2)$ **(ii)** $(5y - z)(25y^2 + 10y + 4)$

5.11) $(m + n + p)(m^2 + n^2 + p^2 - mn - np - pm)$

5.14) **(i)** 1260 **(ii)** –26730

5.15) **(i)** Length $7x - 3$, Breadth $7x - 5$ **(ii)** Length 5y + 5, Breadth $5y + 1$

5.16) **(i)** 3, x and $x - 25$ **(ii)** $4k$, $(2y + 5)$, $(y - 2)$

Miscellaneous Examples of 'Darpan' S7

1) **What should be added in P(y) = $12y^3 - 39y^2 + 50y + 97$ So that the resultant polynomial is divisible by $y + 1$? #**

$$
\begin{array}{r|l}
 & 12y^2 - 51y + 101 \\
\hline
y+1 & 12y^3 - 39y^2 + 50y + 97 \\
 & \underline{12y^3 \pm 12y^2} \\
 & \quad -51y^2 + 50y \\
 & \quad \underline{-51y^2 \mp 51y} \\
 & \qquad\quad 101y + 97 \\
 & \qquad\quad \underline{101y \pm 101} \\
 & \qquad\qquad\quad -4
\end{array}
$$

➤ Here, the remainder is – 4. If we add 4 in it, then the remainder becomes 0 and the resultant polynomial is divisible by $y + 1$.

$\therefore\ p(y) = 12y^3 - 39y^2 + 50y + 97 + 4$

$\therefore\ p(y) = 12y^3 - 39y^2 + 50y + 101$

Now, $p(y)$ is divisible by $y + 1$

2) **If we divide the polynomial $ax^5 - 23x^3 + 47x + 1$ by $x - 2$, the remainder is 7. Find the value of a.**

➤ The polynomial $ax^5 - 23x^3 + 47x + 1$ is divided by x –, 2 The remainder is 7.

$\therefore\ p(x) = ax^5 - 23x^3 + 47x + 1$

➡ Divisor $x - 2 = 0$

$\therefore\ x = 2$

and $p(x) = 7$

$\therefore\ p(2) = 7$

UID : P1-C2-S7-Q1

➥ $p(x) = ax^5 - 23x^3 + 47x + 1 = 7$

$\therefore\ p(2) = a(2)^5 - 23(2)^3 + 47(2) + 1 = 7$

$\therefore\ 32a - 23(8) + 94 + 1 = 7$

$\therefore\ 32a - 184 + 95 = 7$

$\therefore\ 32a - 89 = 7$

$\therefore\ 32a = 7 + 89$

$\therefore\ 32a = 96$

$\therefore\ a = \frac{96}{32} = 3$

$\therefore\ a = 3$

3) Factorise : $(a + b)^3 - (a - b)^3 - 2b$ #

➤ Let $a + b = m$

$a - b = n$

$\underline{-\quad +\quad -}$

$2b = m - n$

$= (a + b)^3 - (a - b)^3 - 2b$

$= m^3 - n^3 - (m - n)$

$= (m - n)\ (m^2 + mn + n^2) - (m - n)$

$= (m - n)\ [m^2 + mn + n^2 - 1]$

$= 2b[(a + b)^2 + (a + b)\ (a - b) + (a - b)^2 - 1]$

$= 2b[(a^2 + 2ab + b^2 + a^2 - b^2 + a^2 - 2ab + b^2) - 1]$

$= 2b\ (3a^2 + b^2 - 1)$

4) Prove that,

$$\frac{\mathbf{0.87 \times 0.87 \times 0.87 + 0.13 \times 0.13 \times 0.13}}{\mathbf{0.87 \times 0.87 - 0.87 \times 0.13 + 0.13 \times 0.13}} = \mathbf{1}$$

➤ $$\frac{(0.87)^3 + (0.13)^3}{(0.87)^2 - 0.87 \times 0.13 + (0.13)^2}$$

Let $a = 0.87,\ b = 0.13$

$$\frac{a^3 + b^3}{(a^2 - ab + b^2)}$$

$$= \frac{(a + b)(a^2 - ab + b^2)}{(a^2 - ab + b^2)}$$

$= a + b$

$= 0.87 + 0.13$

$= 1$

5) Find the value of $(-7)^3 + (12)^3 + (-5)^3$.

➤ Let, $a = -7$

$b = 12$

$c = -5$

➥ $a + b + c = (-7) + (12) + (-5) = -12 + 12 = 0$

➥ If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3\,abc$

$= 3\ (-7) \times 12 \times (-5)$

$= 1260$

6) $a + b = 10$ and $ab = 21$ then find the value of $a^3 + b^3$.

➤ $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (10)^3 - 3\ (21)\ (10)$

$= 1000 - 630$

$= 370$

7) Factorise : $a^6 - 64b^6$

➤ $(a^3)^2 - (8b^3)^2$

$= (a^3 - 8b^3)(a^3 + 8b^3)$

$= (a - 2b)\ (a^2 + 2ab + 4b^2)\ (a + 2b)\ (a^2 - 2ab + 4b^2)$

$= (a - 2b)\ (a + 2b)\ (a^2 + 2ab + 4b^2)\ (a^2 - 2ab + 4b^2)$

8) If $a + b + c = 9$ and $ab + bc + ca = 26$ then find the value of $a^3 + b^3 + c^3 - 3abc$.

➤ $(a + b + c) = 9$

$\therefore\ (a + b + c)^2 = (9)^2$

$\therefore\ a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 81$

$\therefore\ a^2 + b^2 + c^2 + 2\ (ab + bc + ca) = 81$

$\therefore\ a^2 + b^2 + c^2 + 2\ (26) = 81$

$\therefore\ a^2 + b^2 + c^2 + 52 = 81$

$\therefore\ a^2 + b^2 + c^2 = 81 - 52 = 29$

$\therefore\ a^2 + b^2 + c^2 = 29$

➤ $a^3 + b^3 + c^3 - 3abc = (a + b + c)\ [a^2 + b^2 + c^2 - ab - bc - ca]$

$= (9)\ [a^2 + b^2 + c^2 - (ab + bc + ca)]$

$= (9)\ [29 - (26)] = 9(3) = 27$

9) Factorise : $x^2 - \frac{5}{12}x + \frac{1}{24}$

➤ $$\frac{1}{24}\left[24x^2 - 24\left(\frac{5}{12}\right)x + 24\left(\frac{1}{24}\right)\right]$$

$$= \frac{1}{24}[24x^2 - 10x + 1]$$

$$= \frac{1}{24}[24x^2 - 6x - 4x + 1]$$

$$= \frac{1}{24}[6x\ (4x - 1) - 1\ (4x - 1)]$$

$$= \frac{1}{24}[(4x - 1)\ (6x - 1)]$$

UID : P1-C2-S7-Q3

10) $a, b, c \neq 0$, $a + b + c = 0$ **prove that,**

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$$ #

➤ $a + b + c = 0$

$a^3 + b^3 + c^3 = 3abc$ ...(1)

➡ $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$

$$= \frac{abc}{abc}\left[\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right]$$

$$= \frac{a^3 + b^3 + c^3}{abc}$$ ...(2)

➡ Substituting (1) in (2),

$$= \frac{3abc}{abc}$$

$= 3$

11) $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$

➤ $8p^3 + \frac{1}{125} + \frac{12}{5}p^2 + \frac{6}{25}p$

$$\boxed{a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3}$$

$$\therefore (2p)^3 + 3(2p)^2\left(\frac{1}{5}\right) + 3(2p)\left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3$$

$$\therefore \left(2p + \frac{1}{5}\right)^3 = \left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)$$

12) **Prove that** $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$

➤ L.H.S. $= (a + b + c)^3 - a^3 - b^3 - c^3$

$= [(a + b + c)^3 - a^3] - (b^3 + c^3)$

Let $a + b + c = x$

$= [x^3 - a^3] - (b^3 + c^3)$

$= [(x - a)(x^2 + ax + a^2)] - [(b + c)(b^2 - bc + c^2)]$

$= [a + b + c - a][(a + b + c)^2 + a(a + b + c) + a^2] - (b + c)(b^2 - bc + c^2)$

$= [(b + c)][(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca + a^2 + ab + ac + a^2] - (b + c)(b^2 - bc + c^2)$

$= [b + c][3a^2 + b^2 + c^2 + 3ab + 2bc + 3ca] - (b + c)(b^2 - bc + c^2)$

$= (b + c)[3a^2 + b^2 + c^2 + 3ab + 2bc + 3ca - b^2 + bc - c^2]$

$= (b + c)[3a^2 + 3ab + 3bc + 3ca]$

$= (b + c)[3a(a + b) + 3c(a + b)]$

$= (b + c)(a + b)(3a + 3c)$

$= 3(b + c)(a + b)(a + c)$

$= 3(a + b)(b + c)(c + a)$

$\therefore$ L.H.S. = R.H.S.

Very Short Questions of 'Darpan'

MCQ's S8

✧ **Select the proper option from the given alternatives and give the answer.**

Based on Exercise 2.1

1) **A polynomial with zero degree is called .... polynomial.**

(A) Linear (B) Constant

(C) Zero (D) Quadratic

2) **What is the degree of linear polynomial ?**

(A) 1 (B) 2

(C) 3 (D) 4

3) **Out of the following, which is not a polynomial in variable x ?**

(A) $x^2 - 3$ (B) $x - 2\sqrt{x} + 1$

(C) $x^3 + 4x^2$ (D) None of these

4) **Out of the following, which is not a polynomial in variable x ?**

(A) $x + \frac{1}{x}$ (B) $\sqrt{x} + 5$

(C) $x^3 + x^{-2} + 5$ (D) None of these

5) **The degree of the polynomial $5x^2 - 8x + x^3 + 9$ is ....**

(A) 5 (B) -8 (C) 2 (D) 3

Ans. : Ex. 2.1 (1 – B) (2 – A) (3 – B) (4 – D) (5 – D)

UID : P1-C2-S7-Q10

6) The co-efficient of first term of the polynomial $-3x^2 + 4x^3 - 8x + 12$ is .... #

(A) -3 (B) 4 (C) 12 (D) -8

7) Out of the following .... is a polynomial in variable x.

(A) $x^3 + x^{-1} - 8$ (B) $x^{\frac{1}{2}} + x + 3$

(C) $x^2 - x + \frac{5}{2}$ (D) $x - x^{-5} + 1$

8) $p(x) = 5 - 10x$ is a .... polynomial

(A) Quadratic (B) Cubic

(C) Linear (D) Fifth degree

9) The general form of a linear polynomial in one variable is ....

(A) $p(x) = ax + b$ where $a \neq 0$, $a, b \in R$

(B) $p(x) = ax + b$ where $a = 0$, $a, b \in R$

(C) $p(x) = ax + b$ where $a = 0$, $a, b \in Z$

(D) $p(x) = ax + b$ where $a = 0$, $a, b \in N$

10) In a polynomial with variable x, the exponent of x is ....

(A) Negative (B) Fraction

(C) Positive integer (D) Negative fraction

11) The polynomial with degree 2 is called .... polynomial.

(A) Linear (B) Cubic

(C) Quadratic (D) Fourth degree

12) In $p(x) = x^5 + 8x^3 - 3x^2 + 4x - 12$, the coefficient of x^4 is ....

(A) 1 (B) 0 (C) 4 (D) 8

Based on Exercise 2.2

13) $p(x) = x^3 + 2x^2 + 6x + 5$ then $p(-1)$ ....

(A) 0 (B) -1 (C) 1 (D) 2

$p(x) = x^3 + 2x^2 + 6x + 5$

$p(-1) = (-1)^3 + 2(-1)^2 + 6(-1) + 5$

$= (-1) + 2(1) + 6(-1) + 5$

$= -1 + 2 - 6 + 5$

$= 7 - 7$

$\therefore p(-1) = 0$

(**Remember :** If the base is negative and the power is even number then it becomes positive. If the base is negative and the power is odd number then it becomes negative)

14) $p(x) = 2x^3 - 13x^2 + 17x + 12$ then $p(2) =$ ....

(A) -8 (B) 5 (C) -10 (D) 10

15) The value of $p(x) = 3x^3 - 2x^2 + 5x + 4$ at $x = -1$ is ....

(A) 4 (B) -6 (C) 6 (D) 0

16) The zero of the polynomial $p(x) = 5x + 2$ is ....

(A) $\frac{5}{2}$ (B) $-\frac{5}{2}$ (C) $\frac{2}{5}$ (D) $-\frac{2}{5}$

Hints : $p(x) = 5x + 2 = 0$

$\therefore 5x = -2$

$\therefore x = -\frac{2}{5}$

17) If $p(3) = 0$ then .... is factor of $p(x)$.

(A) $x - 3$ (B) $x + 3$ (C) $x - 2$ (D) $3x + 1$

18) The zero of $7x - 3$ is ....

(A) $\frac{-3}{7}$ (B) $\frac{3}{7}$ (C) $\frac{7}{3}$ (D) $-\frac{7}{3}$

Based on Exercise 2.3

19) When a polynomial $p(x) = x^3 + x^2 - x - 1$ is divided by $x - 1$ the remainder is ....

(A) 1 (B) 0 (C) -1 (D) 2

Hints : The root of $x - 1 = 0$ is $x = 1$

$p(x) = x^3 + x^2 - x - 1$

$\therefore p(1) = (1)^3 + (1)^2 - 1 - 1$

$= 2 - 2 = 0$

$\therefore$ The remainder is 0

20) If $x + 4$ is a divisor of $p(x) = x^3 - 60x + 18$ then find the remainder.

(A) 1 (B) -1 (C) 194 (D) 149

21) If a polynomial $x^3 - 6x^2 + 5x + 5$ is divided by $x - 5$ the remainder is ....

(A) 5 (B) -5 (C) 1 (D) 0

22) If $6x^2 + 5x - 8$ is divided by $2x + 3$, find the remainder.

(A) -2 (B) 2 (C) 1 (D) -1

Ans. : (6 – B) (7 – C) (8 – C) (9 – A) (10 – C) (11 – C) (12 – B) Ex. 2.2 (13 – A) (14 – D) (15 – B) (16 – D) (17 – A) (18 – B) Ex. 2.3 (19 – B) (20 – C) (21 – A) (22 – A)

UID : P1-C2-S8-Q6

Hints : $2x + 3 = 0$

$\therefore\ 2x = -3$

$\therefore\ x = -\frac{3}{2}$

$p(x) = 6x^2 + 5x - 8$

$p\left(-\frac{3}{2}\right) = 6\left(-\frac{3}{2}\right)^2 + 5\left(-\frac{3}{2}\right) - 8$

$= 6\left(\frac{9}{4}\right) + 5\left(-\frac{3}{2}\right) - 8$

$= \frac{54}{4} + \frac{-15}{2} - 8$

$= \frac{54}{4} - \frac{15}{2} - \frac{8}{1}$

$= \frac{54 - 30 - 32}{4}$

$= \frac{54 - 62}{4}$

$= \frac{-8}{4} = -2$

23) **$x^{51} + 51$ is divided by $x + 1$ the remainder is .... #**

(A) 0 (B) 1 (C) 49 (D) 50

24) **If $p(x) = x^3 + 4x^2 + ax + 3$ is divisible by $x + 3$ then the value of a is ....**

(A) -4 (B) 2 (C) -2 (D) 4

Hints : $p(x)$ is divisible by $x + 3$

$\therefore\ x + 3 = 0$

$\therefore\ x = -3$

➥ $p(x) = x^3 + 4x^2 + ax + 3$

$p(-3) = (-3)^3 + 4(-3)^2 + a(-3) + 3$

$= -27 + 4(9) - 3a + 3$

$= -27 + 36 - 3a + 3$

$p(-3) = -3a + 12$

So $p(-3) = 0$

$\therefore\ -3a + 12 = 0$

$\therefore\ -3a = -12$

$\therefore\ 3a = 12$

$\therefore\ a = \frac{12}{3} = 4$

$\therefore\ a = 4$

Based on Exercise 2.4

25) **Which are the factors of $2x^2 + 3x + 1$?**

(A) $(x + 1)(x + 3)$ (B) $(x + 1)(2x + 1)$
(C) $(2x + 1)(3x + 1)$ (D) $(x + 1)(x + 3)$

Hints : $2x^2 + 3x + 1$

$= 2x^2 + 2x + x + 1$

$= 2x(x + 1) + 1(x + 1)$

$= (x + 1)(2x + 1)$

26) **To factorise $2x^2 + x - 6$, the parts of middle term are ....**

(A) $3x, -4x$ (B) $2x, -6x$
(C) $4x, -3x$ (D) $6x, -2x$

27) **What are the factors of $a^3 - 1$?**

(A) $(a - 1)(a + 1)$ (B) $(a + 1)(a^2 - a + 1)$
(C) $(a - 1)(a^2 + a + 1)$ (D) $(a - 1)(a - 1)(a - 1)$

Hints : use the identify,

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a^3 - 1^3 = (a - 1)(a^2 + a + 1)$

28) **$(x - 1)$ is a factor polynomial ?**

(A) $2x^3 - x^2 + x - 2$ (B) $3x^3 + 3x^2 + 6x + 6$
(C) $x^3 + x^2 - x - 2$ (D) $2x^3 - 2x^2 + 2x + 1$

Hints : If $x - 1$ is a factor of then $p(1) = 0$.

29) **The factors of $x^2 + 10x + 21$ is ....**

(A) $(x + 3)(x + 7)$ (B) $(x - 3)(x - 7)$
(C) $(x - 3)(x + 7)$ (D) $(x + 3)(x - 7)$

30) **If .... is added in $x^3 - 76$ then it is divisible by $x - 4$.**

(A) 5 (B) -5 (C) 12 (D) -12

Hints : $x - 4 = 0$

$\therefore\ x = 4$

$p(x) = x^3 - 76$

$= (4)^3 - 76$

$\therefore\ p(x) = 64 - 76 = -12$

To remove -12, we add 12 and then the remainder will become 0.

31) **If one of the factor of $25x^2 - 49y^2$ is $(5x - 7y)$ then the other factor is ....**

(A) $7x + 5y$ (B) $-7x - 5y$
(C) $5x + 7y$ (D) $-5x + 7y$

32) **One of the zero of the polynomial $p(x) = x^3 - 2x^2 + 7x - 6$ is ....**

(A) 0 (B) 1 (C) 2 (D) 3

Ans. : (23 – D) (24 – D) Ex. 2.4 (25 – B) (26 – C) (27 – C) (28 – A) (29 – A) (30 – C) (31 – C) (32 – B)

UID : P1-C2-S8-Q23

Hints : $p(x) = x^3 - 2x^2 + 7x - 6$

The sum of coefficient $1 - 2 + 7 - 6 = 0$

$\therefore$ By remainder theorem $x - 1$ is a factor

$x - 1 = 0$

$\therefore$ $x = 1$

Based on Exercise 2.5

33) If $a + b = 7$. $ab = 5$ then $a^3 + b^3$ = ... #

(A) 138 (B) 832 (C) 831 (D) 238

Hints : $a^3 + b^3 = (a + b)^3 - 3ab\ (a + b)$

$= (7)^3 - 3(5)\ (7)$

$= 343 - 105$

$a^3 + b^3 = 238$

34) If $a - b = 8$, $ab = 3$ then $a^3 - b^3$ = ...

(A) 584 (B) 548 (C) 845 (D) 854

Hints : $a^3 - b^3 = (a - b)^3 + 3ab\ (a - b)$

$= (8)^3 + 3(3)\ (8)$

$= 512 + 9 \times 8$

$= 512 + 72$

$a^3 - b^3 = 584$

35) The value of $5^3 - 3^3 - 2^3$ is ....

(A) 90 (B) 9 (C) 100 (D) 0

Hints : $a + b + c = (5) + (-3) + (-2) = 0$

$a + b + c = 0$ then $a^3 + b^3 + c^3 = 3\ abc$

$\therefore 5^3 - 3^3 - 2^3 = 3\ (5)\ (3)\ (2)$

$= 90$

36) $a - b = 2$, $ab = 3$ then $a^3 - b^3$ = ...

(A) 8 (B) 27 (C) 26 (D) 6

37) If $a = b = c$ then $a^3 + b^3 + c^3 - 3abc$ = ...

(A) a^3 (B) $2a^3$ (C) $3a^3$ (D) 0

Hints : $a = b = c$ (Given) $a^3 + b^3 + c^3 - 3\ abc$

$\therefore a^3 + a^3 + a^3 - 3(a)(a)\ (a)$

$\therefore 3a^3 - 3a^3 = 0$

38) If $x + y + z = 0$ then $x^3 + y^3 + z^3$ is ....

(A) 0 (B) xyz (C) $3xyz$ (D) $2xyz$

39) If $a + b = 5$, $ab = 6$ then $a^3 + b^3$ = ...

(A) 53 (B) 35 (C) – 35 (D) – 53

Hints : $a + b = 5$

$\therefore (a + b)^2 = (5)^2$

$\therefore a^2 + 2ab + b^2 = 25$

$\therefore a^2 + 2(6) + b^2 = 25$

$\therefore a^2 + b^2 = 25 - 12$

$\therefore a^2 + b^2 = 13$

$a^3 + b^3 = (a + b)\ (a^2 - ab + b^2)$

$= (5)\ (13 - 6)$ $(\because a^2 + b^2 = 13)$

$= (5)\ (7)$

$= 35$

40) The value of $(-7)^3 + (12)^3 + (-5)^3$ is ....

(A) 1260 (B) 1620 (C) 1206 (D) 6120

Hints : If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$.

$\therefore (-7)^3 + (12)^3 + (-5)^3 = 3\ (-7)\ (12)\ (-5)$

$= (-21)\ (-60)$

$= 1260$

Ans. : Ex. 2.5 (33 – D) (34 – A) (35 – A) (36 – C) (37 – D) (38 – C) (39 – B) (40 – A)

Very Short types questions S9

✧ **Answer the following questions in short :**

1) If $x^2 + \frac{1}{x^2} = 79$ then find the value of $x - \frac{1}{x}$.

2) If $x + \frac{1}{x} = 3$ then find the value of $x^6 + \frac{1}{x^6}$.

3) If $a + b + c = 9$ and $ab + bc + ca = 23$ then find the value of $a^2 + b^2 + c^2$.

4) If $a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}} = 0$ then find the value of $(a + b + c)^3$.

5) If $x - \frac{1}{x} = \frac{15}{4}$ then find the value of $x + \frac{1}{x}$.

6) The volume of a cuboid is $3x^2 - 27$. Find its dimension.

7) Find the value of $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$.

8) Find the factor of $x^2 - 1 - 2a - a^2$.

9) If $x + 1$ is a factor of $x^3 + a$ then find the value of a.

10) $f(x) = x^4 - 2x^3 + 3x^2 - ax - b$ is divided by $x - 1$, the remainder is 6. Find the value of $a + b$.

11) $x - 2$ and $x - \frac{1}{2}$ are factor of $px^2 + 5x + r$ obtain the relation between p and r.

UID : P1-C2-S8-Q33

12) For the polynomial $f(x)$, $f\left(-\frac{1}{2}\right) = 0$. Which is the factor of $f(x)$? #

Ans. : **(1)** $\sqrt{77}$ **(2)** 322 **(3)** 35 **(4)** $27abc$ **(5)** $\frac{17}{4}$
(6) 3, $x + 3$, $x - 3$ **(7)** $-\frac{5}{12}$
(8) $(x - a - 1)(x + a + 1)$ **(9)** 1 **(10)** –4
(11) $p = r$ **(12)** $2x + 1$

Fill in the blanks S10

✧ **Fill in the blanks :**

1) $x + \frac{1}{x} = 4$ then $x^4 + \frac{1}{x^4} =$ ....

2) $x^3 - \frac{1}{x^3} = 14$ then $x - \frac{1}{x} =$ ....

3) If $a + b + c = 0$ then $\frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} =$ ....

4) $(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2) =$ ....

5) $\frac{a}{b} + \frac{b}{a} = 1$ then $a^3 + b^3 =$ ....

6) If $3x + \frac{2}{x} = 7$ then $9x^2 - \frac{4}{x^2} =$ ....

7) If $x^3 - 3x^2 + 3x - 7 = (x + 1)(ax^2 + bx + c)$ then $a + b + c =$ ....

8) The factors of $x^4 + 4$ is ....

9) $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^2$ then $k =$ ....

10) The value of $\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$ is ....

11) $(x + y)^3 - (x - y)^3 =$ ....

12) If $a^2 + b^2 + c^2 = 25$ and $ab + bc + ca = 3$ then $a + b + c =$ ....

13) $x + 1$ is a factor of $x^n + 1$ then $n =$ ....

14) If $x^{140} + 2x^{151} + k$ is divisible by $x + 1$ then $k =$ ....

15) If one of the factor of other $x^4 + x^2 - 20$ is $x^2 + 5$ is ....

16) If $x^2 + x + 1$ is a factor $3x^3 + 8x^2 + 8x + 3 + 5k$ then the value of k is ....

17) If $(3x - 1)^7 = a_7x^7 + a_6x^6 + a_5x^5 + ... + a_1x + a_0$ then $a_0 + a_1 + a_2 + ... + a_6 + a_7 =$ ....

18) If $(x - 1)$ is a factor of $f(x)$ but it is not a factor of $g(x)$ then $(x - 1)$ is a factor of ....
(a) $f(x)\,g(x)$ (b) $-f(x) + g(x)$
(c) $f(x) - g(x)$ (d) $[f(x) + g(x)]g(x)$

19) The area of the rectangle is $25a^2 - 35a + 12$. Then its dimensions are ....

20) If $3x = a + b + c$ then $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c) =$ ....

Ans. : **(1)** 194 **(2)** 2 **(3)** 3 **(4)** $a^6 - b^6$ **(5)** 0 **(6)** 35 **(7)** 4 **(8)** $(x^2 + 2x + 2)(x^2 - 2x + 2)$ **(9)** 8 **(10)** 2 **(11)** $2y(3x^2 + y^2)$ **(12)** ±16 **(13)** n odd integer **(14)** 1 **(15)** $x^2 - 4$ **(16)** $\frac{2}{5}$ **(17)** 128 **(18)** (a) $f(x)\,g(x)$ **(19)** $(5a - 4)(5a - 3)$ **(20)** 0

Illustrations of Text book for practice S11

Ex. 1 : Find the degree of each of the polynomials given below :
(i) $x^5 - x^4 + 3$ **(Ans. 5)**
(ii) $2 - y^2 - y^3 + 2y^8$ **(Ans. 8)**
(iii) 2 **(Ans. 0)**

Ex. 2 : Find the value of each of the following polynomials at the indicated value of variables :
(i) $p(x) = 5x^2 - 3x + 7$ at $x = 1$ **(Ans. 9)**
(ii) $q(y) = 3y^3 - 4y + \sqrt{11}$, at $y = 2$ $\left(\textbf{Ans. } 16 + \sqrt{11}\right)$
(iii) $q(t) = 4t^4 + 5t^3 - t^2 + 6$ at $t = a$
(Ans. $4a^4 + 5a^3 - a^2 + 6$)

Ex. 3 : Check whether –2 and 2 are zeroes of the polynomial $x + 2$.
(Ans. –2 is a zero or the polynomial $x + 2$, but 2 is not)

Ex. 4 : Find a zero of the polynomial $p(x) = 2x + 1$.
$\left(\textbf{Ans. } x = -\frac{1}{2}\right)$

Ex. 5 : Verify whether 2 and 0 are zeroes of the polynomial $x^2 - 2x$. **(Ans. ($p(2) = 0$ and $p(0) = 0$)**

Ex. 6 : Divide $p(x)$ by $g(x)$, where $p(x) = x + 3x^2 - 1$ and $g(x) = 1 + x$. **(Ans. Quotient : $3x - 2$, Remainder : 1)**

Ex. 7 : Divide the polynomial $3x^4 - 4x^3 - 3x - 1$ by $x - 1$.
(Ans. Quotient : $3x^3 - x^2 - x - 4$, Remainder : –5)

UID : P1-C2-S9-Q12

Ex. 8 : Find the remainder obtained on dividing $p(x) = x^3 + 1$ by $x + 1$. # **(Ans. 0)**

Ex. 9 : Find the remainder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by $x - 1$. **(Ans. 2)**

Ex.10 : Check whether the polynomial $q(t) = 4t^3 + 4t^2 - t - 1$ is a multiple of $2t + 1$. **(Ans. $q(t)$ is a multiple of $2t + 1$ because the remainder obtained is 0)**

Ex.11 : Examine whether $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$ and $2x + 4$. **(Ans. Yes, $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$ and $2x + 4$)**

Ex.12 : Find the value of k, if $x - 1$ is factor of $4x^3 + 3x^2 - 4x + k$. **(Ans. $k = -3$)**

Ex. 13 : Factorise $6x^2 + 17x + 5$ by splitting the middle term, and by using the factor theorem. **(Ans. $(3x + 1)(2x + 5)$)**

Ex. 14 : Factorise $y^2 - 5y + 6$ by using the factor theorem. **(Ans. $(y - 2)(y - 3)$)**

Ex. 15 : Factorise $x^3 - 23x^2 + 142x - 120$ **(Ans. $(x - 1)(x - 10)(x - 12)$)**

Ex. 16 Find the following products using appropriate identities :

(i) $(x + 3)(x + 3)$ **(Ans. $x^2 + 6x + 9$)**

(ii) $(x - 3)(x + 5)$ **(Ans. $x^2 + 2x - 15$)**

Ex. 17 : Evaluate 105×106 without multiplying directly. **(Ans. 11130)**

Ex. 18 : Factorise :

(i) $49a^2 + 70ab + 25b^2$ **(Ans. $(7a + 5b)(7a + 5b)$)**

(ii) $\frac{25}{4}x^2 - \frac{y^2}{9}$ **(Ans. $\left(\frac{5}{2}x + \frac{y}{3}\right)\left(\frac{5}{2}x - \frac{y}{3}\right)$)**

Ex. 19 : Write $(3a + 4b + 5c)^2$ in expanded form. **(Ans. $9a^2 + 16b^2 + 25c^2 + 24ab + 40bc + 30ac$)**

Ex. 20 : Expand $(4a - 2b - 3c)^2$. **(Ans. $16a^2 + 4b^2 + 9c^2 - 16ab + 12bc - 24ac$)**

Ex. 21 : Factorise $4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$. **(Ans. $2x - y + z)^2 = (2x - y + z)(2x - y + z)$)**

Ex. 22 : Write the following cubes in the expanded from :

(i) $(3a + 4b)^3$ **(Ans. $27a^3 + 64b^3 + 108a^2b + 144ab^2$)**

(ii) $(5p - 3q)^3$ **(Ans. $125p^3 - 27q^3 - 225p^2q + 135\ pq^2$)**

Ex. 23 : Evaluate each of the following using suitable identities :

(i) $(104)^3$ **(Ans. 1124864)**

(ii) $(999)^3$ **(Ans. 997002999)**

Ex. 24 : Factorise : $8x^3 + 27y^3 + 36x^2y + 54xy^2$ **(Ans. $(2x + 3y)(2x + 3y)(2x + 3y)$)**

Ex. 25 : Factorise : $8x^3 + y^3 + 27z^3 - 18xyz$ **(Ans. $(2x + y + 3z)(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6xz)$)**

Questions from Module

Higher Order Thinking Skills Examples S13

1) If $p(x) = x^2 - 2\sqrt{2}\,x + 1$ then find $p(2\sqrt{2})$.

2) If $x + y + z = 9$ and $xy + yz + zx = 26$, then find $x^2 + y^2 + z^2$.

3) Factorise : $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$.

4) If $a + b + c = 0$ then prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$ when a, b, c are not zero together.

5) Prove that : $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$

Answers

1) 2

2) 29

3) $\left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)$.

UID : P1-C2-S11-Q8

Practice S14

1. Fill in the blanks by selecting proper option from the given alternatives (Each has 1 mark.) (5)

1) **The coefficient of x^2 in the polynomial $p(x) = \frac{\pi}{2}x^2 + x$ is .... #**

(A) 1 (B) 0 (C) $\frac{\pi}{2}$ (D) π

2) **If $p(x) = 5x^3 - 2x^2 + 3x - 2$, then $p(1) =$ ....**

(A) 2 (B) 4 (C) 8 (D) –4

3) **If $p(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$ and $a + c = b + d$ then common factor of $p(x)$ is ....**

(A) $(x - 1)$ (B) $(x + 1)$ (C) $(x - 2)$ (D) $(x + 2)$

4) **The zero of $p(x) = \sqrt{3}x - 3$ is ....**

(A) $-\sqrt{3}$ (B) $+\sqrt{3}$ (C) 3 (D) –3

5) **$x - 1$ is a factor of $p(x) = x^2 + x + k$ then $k =$ ....**

(A) –2 (B) 2 (C) 3 (D) –3

2. Fill in the blanks (Each has 1 mark) (5)

1) If $a + b + c = 0$ then $a^3 + b^3 + c^3 =$ ....

2) $(x - y)^3 =$ ....

3) The degree of $p(x) = 5x^3 + 4x^2 + 7x$ is ....

4) If $p(x) = (x - 1)(x + 1)$ then $p(2) =$ ....

5) When $p(x) = x^3 + 1$ is divided by $x + 1$, the remainder is ....

3. Say whether the following statement is true or false. (Each has 1 mark) (5)

1) $p(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$. If $a + b + c + d = 0$ then $x + 1$ is a factor of $p(x)$.

2) $p(x) = 3x - 2$ has two zeroes.

3) The polynomial $p(t) = 4t^3 + 4t^2 - t - 1$ is a multiple of $2t + 1$.

4) $x + 2$ is not a factor of $x^3 + 3x^2 + 5x + 6$.

5) The polynomial with degree n has at most $x + 1$ zeroes.

4. Answer the following questions. (Each 1 mark) (5)

1) If $p(t) = 2 + t + 2t^2 - t^3$ then find $p(1)$.

2) Find the reminder when $x^3 + ax^2 + 6x - a$ is divided by $x - a$.

3) Factorise : $12x^2 - 7x + 1$.

4) Find the value of 105×106.

5) **Match the following :**

	Part - I		Part - II
(i)	$p(x) = x^5 - x^4 + 3$ has degree	(a)	8
(ii)	$p(x) = 2 - y^2 - y^3 + 2y^8$ has degree	(b)	0
		(c)	5
(iii)	The degree of $p(x) = 2$ is	(d)	4

Answers

1. (1 – C) (2 – B) (3 – B) (4 – B) (5 – A)

2. **(1)** $3abc$ **(2)** $x^3 - 3x^2y + 3xy^2 - y^3$ **(3)** 3 **(4)** 3 **(5)** 0

3. **(1)** True **(2)** False **(3)** True **(4)** False **(5)** False

4. **(1)** 4 **(2)** $2a^3 + 5a$ **(3)** $(4x - 1)(3x - 1)$ **(4)** 11130 **(5)** (i – c) (ii – a) (iii – b)

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UID : P1-C2-S14-Q1.1